NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

TWO-SIDED MATCHING FOR THE US NAVY'S ENLISTED DETAILING PROCESS: A COMPARISON OF DEFERRED ACCEPTANCE AND LINEAR PROGRAMMING VIA SIMULATION

by

Joshua H. Ho
Eng Hwee Low

December 2002

Thesis Advisor:	William R. Gates
Associate Advisor:	William D. Hatch II

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REPORT DOCUMENTATION PAGE	*Form Approved OMB No. 0704-0188*

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE December 2002	3. REPORT TYPE AND DATES COVERED Master's Thesis

4. TITLE AND SUBTITLE: Two-Sided Matching for the US Navy's Enlisted Detailing Process : A Comparison of Deferred Acceptance And Linear Programming Via Simulation

5. FUNDING NUMBERS

6. AUTHOR(S) Ho, Joshua H. and Low, Eng Hwee

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING /MONITORING AGENCY NAME(S) AND ADDRESS(ES)
N/A

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(maximum 200 words)*

Recent studies of 2-sided matching mechanisms have suggested potential benefits for implementation into the Navy enlisted assignment process. The proposed matching process improves the chance of commands and sailors being assigned either a sailor or billet of their choice. The same studies focused on a particular two-sided Deferred Acceptance (DA) matching algorithm which ensures stable matches, prevents "off-the-site" trades between the matching parties and upholds integrity of the matching system. Although stable matches are important in a voluntary labor market, the DA algorithm may still favor one party depending on whether the command or sailor biased form of the algorithm is used.

The Linear Programming (LP) algorithm is an alternative that could optimize system (command and sailor) effectiveness and promote a balanced approach to meeting the preferences of both parties. Although LP does not guarantee stable matches, it is still employed by selective British hospitals for their matching with interns. The extent of the unstable matches has not been examined to measure it against the benefit of higher system effectiveness. This thesis will evaluate if the LP algorithm could serve as a better alternative to DA algorithm through simulation of the Navy enlisted assignment process.

14. SUBJECT TERMS
Simulation Model, Distribution and Assignment, Two-Sided Matching, Detailing, Personnel Policy Analysis, Placement

15. NUMBER OF PAGES 106

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

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Joshua H. Ho
Lieutenant Colonel, Republic of Singapore Navy
M.A., Cambridge University, 1993

Eng Hwee Low
Civilian, Ministry of Defence, Singapore
B.A.(Hons), National University of Singapore, 1992

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MANAGEMENT

from the

NAVAL POSTGRADUATE SCHOOL
December 2002

Authors: Joshua H. Ho

Eng Hwee Low

Approved by: William R. Gates
Thesis Advisor

William D. Hatch II
Associate Advisor

Douglas A. Brook
Dean, Graduate School of Business and Public Policy

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ABSTRACT

Recent studies of 2-sided matching mechanisms have suggested potential benefits for implementation into the Navy enlisted assignment process. The proposed matching process improves the chance of commands and sailors being assigned to a party of choice. The same studies focused on a particular two-sided Deferred Acceptance (DA) matching algorithm which ensures stable matches, prevents "off-the-site" trades between the matching parties and upholds integrity of the matching system. Although stable matches are important in a voluntary labor market, the DA algorithm may still favor one party depending on whether the command or sailor biased form of the algorithm is used.

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ACKNOWLEDGMENTS

We would like to express our heartfelt thanks to both our advisers, Professor William Gates and CDR William Hatch who had provided invaluable advise, comments to our thesis and pointing us in the right direction. They have also contributed in many other ways through discussions on subjects related to military manpower and matters relating to our curriculum. We are indeed very grateful to them and hope that we can continue to maintain the friendships that we have established during the course of the thesis.

We would also like to express our gratitude to our ever supportive wives and children as they had to compete for time with their husbands and daddies as a result of demand from the school schedule and the thesis. Words are not sufficient in expressing the love that we feel for them. Joshua would like to express his undying love to Jessie Thong (wife) and Christopher Ho (son) and Eng Hwee would like to express his affection to Goh Lay Ling (wife) and Seth Low (child).

Last but not least, we like to thank our fellow students at NPS who had made our stay memorable and for assisting us in the rest of the curriculum as we churned out the thesis and of course to our respective organizations for having selected us to come to this most wonderful Institution.

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I. INTRODUCTION

A. BACKGROUND

At the Second Annual Navy Workforce Research and Analysis Conference held from the 14 to 15 February 2002, the Assistant Secretary of the Navy (M&RA), Hon. William A. Navas, Jr., laid out the agenda for Navy Manpower, Personnel and Training (MPT) Research and Development. He stressed that people were the critical multiplier for readiness and capability, underlined by the fact that three-fifths of DON's budget is catered to the manpower, personnel and training system. With limited resources available, trade-offs must be made and success rests on developing an expert knowledge of the complex Manpower, Personnel and Training processes. Mr Navas highlighted that the DON must be able to:

- Effectively use the total force – that is, requiring fewer people and making better acquisition /design decisions with people in mind,
- Improve its ability to recruit and retain the right people for the right jobs,
- Improve the return from its training investment,
- Improve force management,
- Optimize compensation and benefits, and
- Solve organizational issues that limit DON's ability to plan and manage effectively.

In order to achieve success on these six key areas, Mr Navas elaborated on the need to incorporate more flexible career management with customer-centered processes, maximize sailor choice and measure performance of both people and systems.

In response to the agenda set by the Assistant Secretary of the Navy (M&RA), the Commander, Navy Personnel Command (NPC), RADM Jacob Shuford, outlined NPC's strategic priorities at the same conference as:

- “Project Success,”
- Sailor Relationship Management, and
- Strategic IT and R&D investment.

Specifically for the “success of existing projects,” RADM Shuford indicated that sailor career management would undergo a transformational change with the introduction of a web-based marketplace for enlisted distribution and assignment. However, before full-scale implementation of the web-based marketplace occurs, a feasibility study will be conducted with an E-9 detailing pilot in October 2002. The E-9 detailing pilot culminates a review of the current enlisted distribution process and its subsequent redesign currently being undertaken by the Naval Personnel Research, Studies, and Technology (NPRST) in partnership with the Naval Postgraduate School (NPS), the University of Memphis and the University of Mississippi. The web-based market place will incorporate both sailor and command preferences in an automated two-sided matching mechanism.

Two-sided matching mechanisms have been used in the entry-level labor market for new physicians, organized via the National Resident Matching Program (NRMP), since the 1950s, and by the labor market for British medical interns and hospitals since the 1970s. Whilst the Deferred Acceptance (DA) algorithm was used for matching in the NRMP, the British markets have used three categories of matching algorithms: the priority, deferred acceptance and linear programming (LP) algorithms. Of the three algorithms used, the priority algorithm used in Newcastle, Sheffield, Birmingham and Edinburgh was abandoned due to unstable matches and an increase in the number of applicants submitting only one choice. Currently two matching mechanisms survive in Britain: the LP algorithm used in London and Cambridge and the DA algorithm used in Edinburgh. These two matching mechanisms can possibly be used as the matching mechanism in the U.S. Navy’s enlisted detailing process.

B. PURPOSE

This thesis will evaluate the use of Linear Programming as an alternative two-sided matching algorithm and compare its performance against the DA algorithm in the U.S. Navy's enlisted distribution market. Performance measures will be developed and the matching results evaluated with these measures, through simulation of the two-sided matching process. Whilst the DA algorithm guarantees a stable match (Roth, 1990), the "optimal" match favors only one side of the labor market, depending on which party initiated the matching process; that is, the command biased DA algorithm favors the commands, and vice versa. Although the LP algorithm does not eliminate unstable matches, it can provide a more balanced approach, ending up with an assignment that caters to both parties. The number of unstable matches that results may not be very large.

Past simulation studies on the two-sided DA algorithm had been used with estimated preference factors (Ng and Soh, 2001). The same simulation also assumed a Cobb-Douglas utility function (i.e., interactions amongst the preference factors) to derive the preference list. A survey was conducted on the Aviation Support Equipment Technician (AS) rating (Butler and Molina, 2002) to determine a more realistic set of preference factors. This thesis will evaluate the benefits and shortfalls of the LP algorithm using a revised Utility function, while incorporating the more realistic/representative preference factors using Molina & Butler's findings.

C. RESEARCH QUESTIONS

1. Primary Research Question

What is the relative performance of the Linear Programming and the Deferred Acceptance matching algorithm when applied to the U.S. Navy's enlisted assignment process?

2. Secondary Research Questions

- What are the measures of success for two-sided matching regardless of matching mechanisms used?
- What is a more robust simulation model to evaluate the performance of Linear Programming vice the Deferred Acceptance matching mechanism?

D. SCOPE AND LIMITATION

1. Scope

The scope of the thesis will include:

- An overview of the current USN enlisted assignment process and a discussion of its shortcomings.
- A review of the existing simulation program, its characteristics and a description of the enhancements required.
- An enhancement of the existing simulation program for use in two-sided matching.
- A review of the matching mechanisms: specifically the Linear Programming and the Deferred Acceptance mechanisms.
- A description of the performance measures used to evaluate the two matching mechanisms.
- A series of simulations of the two-sided matching process using both the Linear Programming and Deferred Acceptance matching mechanisms.
- An evaluation of the performance of the Linear Programming vice the Deferred Acceptance matching mechanism from the results of the simulation.

2. Limitation

This study will only simulate the U.S. Navy enlisted detailing process and compare the performance of the Linear Programming vice the Deferred Acceptance matching mechanism in conducting two-sided matching. Thus, the thesis will not cover other matching mechanisms nor will it cover the officer community.

E. METHODOLOGY

The methodology used in the thesis will consist of the following steps:

- Conduct a literature search of books, magazine articles, CD-ROM systems and other library information resources.
- Review the current U.S. Navy manpower planning process focusing on the area of distribution and assignment (detailing).
- Review the Linear Programming and Deferred Acceptance matching mechanisms.
- Review the existing software program developed for simulation of two-sided matching.
- Revise the simulation program.
- Conduct simulations with both the Linear Programming and Deferred Acceptance matching mechanisms.
- Detail the results of the simulation and conduct an analysis of the results.
- Obtain conclusions from the simulation results.

F. EXPECTED BENEFITS OF THE STUDY

Research is being done by NPRST to determine how two-sided matching can be implemented in the Navy. This thesis will provide additional information on the effects of two-sided matching using actual sailor and command preferences, and evaluate whether Linear Programming or the Deferred Acceptance matching mechanism would be more

appropriate for use. The results from these simulations can then be compared subsequently with actual results from the trial.

G. ORGANIZATION OF THE THESIS

The thesis will be organized as follows:

- Chapter II will provide an overview of the current U.S. enlisted assignment process.
- Chapter III will provide an overview of the existing simulation program and describe the DA algorithm.
- Chapter IV will describe the enhancements made to the simulation program and the LP algorithm.
- Chapter V will detail the findings of the simulations and analyze the results.
- Chapter VI will summarize conclusions, recommendations and areas for future research.

II. OVERVIEW OF THE CURRENT U.S. NAVY ENLISTED ASSIGNMENT PROCESS

A. THE MANPOWER, PERSONNEL AND TRAINING SYSTEM

The United States Navy's Manpower, Personnel and Training system consists of four processes: (i) Manpower Requirements; (ii) Manpower Programming; (iii) Personnel Planning and (iv) Personnel Distribution. The Manpower, Personnel and Training System is depicted in Figure 1.

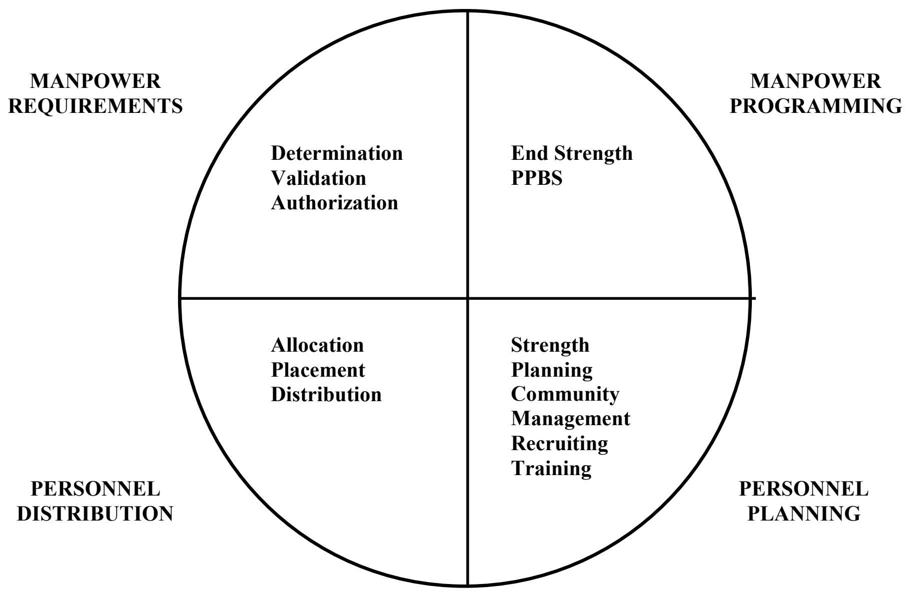

Figure 1. The U.S. Navy Manpower, Personnel and Training System (From: Manpower, Personnel and Training Processes power-point brief by CDR William D. Hatch, June 2001)

This thesis will focus on the personnel distribution process of the Manpower, Personnel and Training System for the active duty Enlisted sailors.

B. THE PERSONNEL DISTRIBUTION PROCESS

The Personnel Distribution Process consists of three sub-processes known as the distribution triad. The three sub-processes are allocation, placement and assignment or detailing. The goal of the personnel distribution process is to get the right sailor with the right training to the right billet at the right time, or "R^4" as it is better known. This section will only highlight the key aspects of the personnel distribution process. A more complete description of the personnel distribution process is given in "Characterizing Sailor and Command Enlisted Placement and Assignment Preferences" (Butler and Molina, 2002).

1. The Allocation Sub-Process

Navy Personnel Command is responsible for the allocation sub-process. Before allocation can be done, the total available personnel (or personnel inventory) must first be separated into distributable inventory and non-distributable inventory. Non-distributable inventory consist of persons in the Transient, Patient, Prisoner or Holdee (TPPH) list, persons who are Awaiting Instruction (AI), Students and persons whose End of Active Obligated Service (EAOS) is less than nine months away. These three categories of people will be put into the Individuals Account (IA). The allocation process then apportions the distributable inventory to the four Manning Control Authorities (MCAs) according to manning priorities established by the Chief of Naval Operations (CNO). The four Manning Control Authorities include Commander-in-Chief, U.S. Pacific Fleet (MCA-P), Commander-in-Chief, U.S. Atlantic Fleet (MCA-L), Commander, Navy Personnel Command (MCA-B), and Commander, Naval Reserve Forces (MCA-R). The categorization of personnel inventory is depicted in Figure 2.

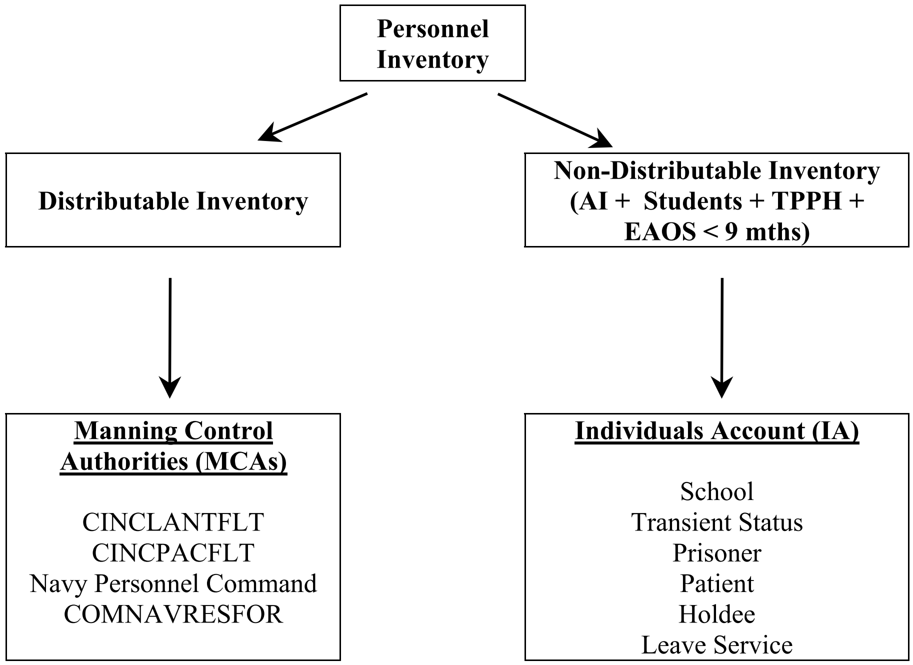

Figure 2. Categorization of Personnel Inventory (After: Manpower, Personnel and Training Processes power-point brief by CDR William D. Hatch, June 2001)

The output of the allocation sub-process is the promulgation of the Navy Manning Plan (NMP), which is a detailed document showing the prioritized allocation of distributable inventory to the various Manning Control Authorities down to the individual commands. The NMP guides the subsequent sub-processes of placement and assignment by specifying the number and characteristics of the sailor (Rate, Rating and Navy Enlisted Classification) that each command will get by indicating if the billets are Priority 1, 2 or 3 or no priority. Priority 1 and 2 billets are rationalized across the MCAs whilst Priority 3 billets are rationalized within the MCAs. The allocation sub-process is summarized in Figure 3.

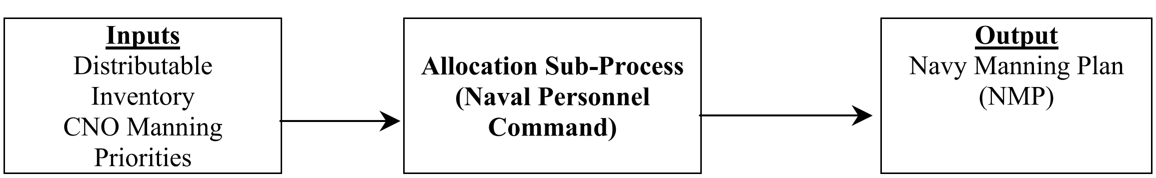

Figure 3. The Allocation Sub-Process

2. The Placement Sub-Process

The Enlisted Placement Management Center (EPMAC) is responsible for the placement sub-process, which is the second leg of the distribution triad. EPMAC acts as the command advocate for the E-5 to E-9 sailors and strives to achieve R^4 by ensuring that the right person, with the proper occupational skills, occupies the right billet on time. The placement officers in EPMAC use the NMP as a reference document to communicate the command's billet requirements to the detailers. Besides executing the NMP, placement officers also have to deal with additional requisitions when MCAs have activities that require a manning above the NMP or when losses are unplanned and differ from those projected in the NMP. The placement officers are in charge of a set of ratings and Navy Enlisted Classifications (NECs) and negotiate with detailers who are more focused on a specific rating or NEC. The placement sub-process is depicted in Figure 4.

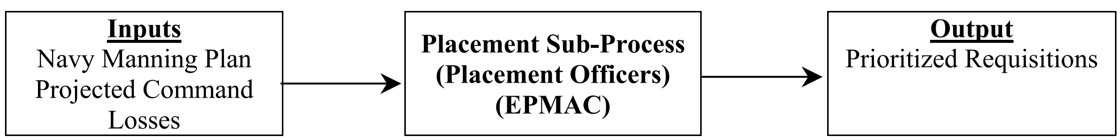

Figure 4. The Placement Sub-Process

3. The Assignment Sub-Process

The assignment sub-process is the third and final leg of the distribution triad and is also known as the detailing sub-process. The detailer is the principal agent in the assignment sub-process and is the sailor advocate. The detailer will match sailors with the necessary skill sets to the prioritized requisitions. The assignment sub-process repeats every two weeks in what is known as the requisition cycle, and typically, each detailer will assign 45 sailors across 60 billets every requisition cycle. During each requisition cycle, detailers will sort through the applications of various sailors and assign the most qualified sailor to the billet.

For each billet, the sailors are short-listed by their 'must have' attributes; that is, characteristics that the sailor must possess before they can be considered for the billet. The 'must have' attributes include the sailor's rate, rating, NEC, skill set vice the billet, gender, projected rotation date (PRD), sea-shore rotation cycles and security classification. If the sailor does not have the skill set for the billet, the detailer can consider sending the sailor to School to acquire the skill set, if there are school house vacancies. After the shortlist, the sailors are then selected for the billet based on considerations which include requisition policies, Navy Manning Plan, fleet balances, Job Advertising and Selection System (JASS) preferences, Permanent Change of Station (PCS) costs, co-location of married couples, and promotion/career opportunities.

Once the detailer has made the assignments, electronic orders are written in the Enlisted Assignment Information System (EAIS). However, before the paper orders are written and sent to the sailor, assignments for sailors of rate E-5 and above are reviewed by EPMAC for billet fit and policy conformance. EPMAC can disapprove orders that fail to meet Fleet readiness, manning and allocation targets. Historically, this has occurred only about 3% of the time. Once the orders are approved, written orders are sent to the sailor.

After the sailors in a requisition cycle are assigned to available billets, new requisitions are uploaded from the EPRES and the detailer releases new billets into JASS, restarting the two-week cycle. Sailors and billets not matched in the cycle are rolled over to the next cycle to be considered again.

The Command Career Counselor (CCC) is the other resident agent in the assignment sub-process and assists the sailors in selecting their 5 most preferred billets and in electronically submitting their preferences into JASS. The CCC will assist the sailors in identifying jobs for which they are qualified by using a combination of training, experience and written manuals. The CCC will also factor in the sailors' desires and personal concerns,

which include, location, career for their spouse and advancement opportunities. The assignment sub-process is depicted in Figure 5.

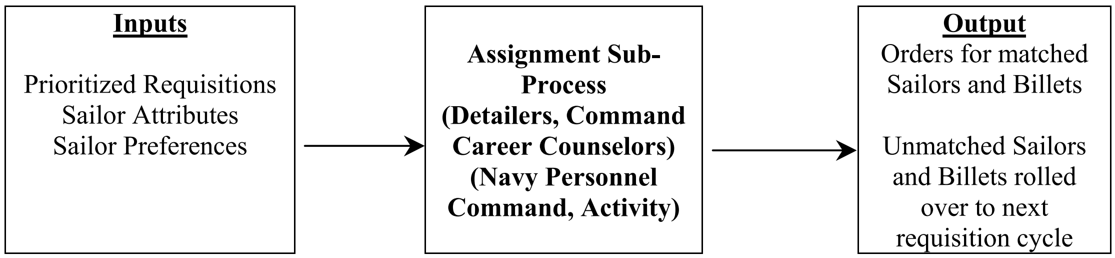

Figure 5. The Assignment Sub-Process

C. SHORTCOMINGS OF THE PERSONNEL DISTRIBUTION PROCESS

From an economics perspective, the allocation and placement sub-processes seek to define the demand for labor; the assignment sub-process seeks to define the supply of labor and, at the same time, clears the labor market by matching sailors and billets. The assignment sub-process is then the critical step since it performs the market clearing function. However, the assignment sub-process is not as effective and efficient as it could be due to several shortcomings.

1. Ineffectiveness

The assignment sub-process is not as effective as it could be as it is manual and relies on the detailer. Consequently, non-optimal assignments can arise and human error is possible which impacts whether the commands get who they want and whether the sailor's needs and preferences are fulfilled. Another consequence is that both sailors and commands perceive the assignment sub-process as subjective.

a. Sub-Optimal Assignments and Human Error

Sub-optimal assignments and human error can result from the high volume of information that the detailer has to consider when making assignments. Firstly, there are the various policies and procedures promulgated by the DoD, CNO, MCAs and the CNPC to

consider. These include factors like PCS cost, Fleet balance, requisition priorities, gapped billets, sea/shore rotation, pay grade, gender and the number of family members. Secondly, there is a need to consider the sailor's 'must have' attributes to see if the sailor has the pre-requisites for the job and then make the assignment based on the sailor's 'should have' attributes. Thirdly, the sailor's preferences will somehow have to be factored into the equation as well.

Besides the volume of information to be processed, non-optimal assignments and human error can occur since assignments depend on the detailer's ability to distinguish between the important 'must have' and 'should have' attributes of the sailor. As this ability is a function of the detailer's knowledge and experience, variability in performance can arise because of the variation in both knowledge and experience of the detailers. There is an indication that this is true as the AS Commands' have listed that the two top reasons for them to intervene in the assignment process is being assigned sailors with the wrong paygrade or wrong NEC (Butler and Molina, 2002).

b. Perception of Subjectivity

Although JASS allows sailors to express up to 5 billet preferences, and command needs are reflected as requisitions in priority order in the allocation and placement sub-processes, sailors and commands still perceive the distribution process as being subjective because both have relinquished the market clearing function to the detailer, an intermediary, instead of performing the function themselves. In fact, sailors believe that they will receive better or different job options by directly contacting the detailer and obtaining insider information (Short, 2000). To make matters worse, there are also no objective performance measures to indicate how well the assignment ranks relative to the preferences of both the sailor and the command.

2. Inefficiency

The assignment sub-process is also not as efficient as it could be because it is manual and relies on the detailer. Specifically, the assignment sub-process is labor intensive and has a long lead time.

a. Labor Intensive and Redundancies

There are about 294 enlisted detailers managing the careers of nearly 330,000 sailors. There are also command career counselors to assist the sailor in making choices and expressing their preferences. Some of these command career counselor billets are collateral duties in the smaller commands, but they are dedicated billets in major commands. As such, there is an overlap between the functions of the detailer and the command career counselor. There are also redundancies built into the orders writing sub-process, where EPMAC clears the orders for sailors of rate E-5 and above to ensure policy conformance. The current process is highly labor intensive and requires a lot of coordination between different groups of professionals. This results in low service quality, where sailors feel frustrated about their ability to access their detailer via the telephone and electronic mail (Butler and Molina, 2002)

b. Long Lead Time

The lead time for the assignment sub-process can be considered long. Sailors will look at or will be considered for assignments nine months prior to their PRD. The whole assignment sub-process can take five to nine months and the sailor can receive written orders anywhere from five months prior to three months after their PRD; the target should be for sailors to receive their orders 6 months prior to their PRD so that there is sufficient time for the sailor to prepare for the change of duty station. The long lead times are a result of the negotiations between the sailor and detailer and between the detailer and placement officer. If the time taken to complete an assignment is reduced, sailors would have sufficient time to schedule and prepare for a move.

D. AN ALTERNATIVE

The effectiveness and efficiency shortcomings in the current detailing sub-process are a result of the need to coordinate and process large amounts of information from different agencies through a central clearing house that is not as aware of the command and sailor preferences as the command and sailor themselves. Thus, a mechanism that allows information to be exchanged between the buyer (command) and the sellers of labor (sailor), and facilitates an objective match of preferences, will improve the effectiveness and efficiency of the detailing sub-process.

Two-sided matching labor markets offer the potential to address the effectiveness and efficiency shortcomings of the current detailing sub-process (Robards, 2001) and the concept of the two-sided matching market was demonstrated to be viable through simulations (Ng and Soh, 2001). However, these studies have focused on a particular two-sided Deferred Acceptance (DA) matching algorithm that ensures stable matches and prevents "off-the-site" trades between the matching parties, thereby upholding the integrity of the matching system. Although stable matches are important in a voluntary labor market, the DA algorithm may still favor one party depending on whether the command or sailor biased form of the algorithm is used.

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III. THE AGENT BASED EMPLOYMENT MARKET SIMULATOR (ABEMS)

A. TWO-SIDED MATCHING

Three different mechanisms have been used in the two-sided matching of medical interns to consultants in Britain (Ünver, 2001). The three algorithms are: (a) Deferred Acceptance (DA), (b) Linear Programming (LP), and (c) Priority. The DA mechanism is predominant in assigning medical interns in the US and UK hospitals, as it generates stable matches. Because the LP algorithm does not guarantee stable matches, it is not as common as the DA mechanism. The LP algorithm is used in Newcastle and Birmingham hospitals. The priority mechanism is no longer in use because of problems that led to hospitals hiring interns up to two years in advance of their graduation dates. Many of the interns also only list a single choice in their applications. A brief description of these three algorithms (using potential matching examples of consultants and interns) is given as follows:

1. Deferred Acceptance (DA)

Assume that a potential match exists between consultants and interns and each consultant and intern have a rank-order list profile Q(f) and Q(ω) respectively. Each consultant, f, starts the process by indicating an interest or "proposing" to his most preferred intern with respect to Q(f). Each intern, ω, temporarily accepts the best consultant's proposal if he is in her rank-order list profile Q(ω), while refusing the remaining candidates. At any other iteration, each consultant, f, who does not have a held offer, proposes to the next best intern in Q(f) who has not yet rejected his proposal. Each intern, ω, holds only the best consultant's proposal with respect to Q(ω) among the one she keeps from the previous step and the new proposal at this iteration, while refusing other candidates. When none of the offers are rejected at a step, the algorithm stops and temporary acceptances become realized matches. Gale and Shapley (1962) first proposed the DA mechanisms using marriage partners for illustration purpose.

Example:

Step 1: f_1, f_2, f_3 and f_5 have no immediate competitors. f_4 and f_6 have to concede their first choices (ω_5 and ω_6) to f_5 and f_2 respectively

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
f_1	ω_3	ω_1	ω_5	ω_4	ω_2	ω_6
f_2	ω_6	ω_1	ω_3	ω_4	ω_5	ω_2
f_3	ω_4	ω_3	ω_6	ω_5	ω_1	ω_2
f_4	$\boxed{\omega_5}$	ω_3	ω_2	ω_6	ω_1	ω_4
f_5	ω_5	ω_1	ω_2	ω_3	ω_6	ω_4
f_6	$\boxed{\omega_6}$	ω_2	ω_5	ω_4	ω_3	ω_1

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
ω_1	f_4	f_3	f_1	f_2	f_5	f_6
ω_2	f_3	f_5	f_6	f_4	f_1	f_2
ω_3	f_5	f_3	f_6	f_2	f_1	f_4
ω_4	f_6	f_4	f_2	f_3	f_1	f_5
ω_5	f_5	f_3	f_2	f_6	f_5	$\boxed{f_4}$
ω_6	f_2	f_3	f_1	$\boxed{f_6}$	f_4	f_5

Step 2: f_4 and f_6 "propose" to second ranked ω_3 and ω_2 respectively

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
f_1	ω_3	ω_1	ω_5	ω_4	ω_2	ω_6
f_2	ω_6	ω_1	ω_3	ω_4	ω_5	ω_2
f_3	ω_4	ω_3	ω_6	ω_5	ω_1	ω_2
f_4	$\boxed{\omega_5}$	$\boxed{\omega_3}$	ω_2	ω_6	ω_1	ω_4
f_5	ω_5	ω_1	ω_2	ω_3	ω_6	ω_4
f_6	$\boxed{\omega_6}$	$\boxed{\omega_2}$	ω_5	ω_4	ω_3	ω_1

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
ω_1	f_4	f_3	f_1	f_2	f_5	f_6
ω_2	f_3	f_5	$\boxed{f_6}$	f_4	f_1	f_2
ω_3	f_5	f_3	f_6	f_2	f_1	$\boxed{f_4}$
ω_4	f_6	f_4	f_2	f_3	f_1	f_5
ω_5	f_5	f_3	f_2	f_6	f_5	$\boxed{f_4}$
ω_6	f_2	f_3	f_1	$\boxed{f_6}$	f_4	f_5

Step 3: f_4 gives way to f_1, "proposes" to third ranked ω_2

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
f_1	ω_3	ω_1	ω_5	ω_4	ω_2	ω_6
f_2	ω_6	ω_1	ω_3	ω_4	ω_5	ω_2
f_3	ω_4	ω_3	ω_6	ω_5	ω_1	ω_2
f_4	$\boxed{\omega_5}$	$\boxed{\omega_3}$	$\boxed{\omega_2}$	ω_6	ω_1	ω_4
f_5	ω_5	ω_1	ω_2	ω_3	ω_6	ω_4
f_6	$\boxed{\omega_6}$	$\boxed{\omega_2}$	ω_5	ω_4	ω_3	ω_1

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
ω_1	f_4	f_3	f_1	f_2	f_5	f_6
ω_2	f_3	f_5	$\boxed{f_6}$	$\boxed{f_4}$	f_1	f_2
ω_3	f_5	f_3	f_6	f_2	f_1	$\boxed{f_4}$
ω_4	f_6	f_4	f_2	f_3	f_1	f_5
ω_5	f_5	f_3	f_2	f_6	f_5	$\boxed{f_4}$
ω_6	f_2	f_3	f_1	$\boxed{f_6}$	f_4	f_5

Step 4: f_4 gives way to f_6, "proposes" to fourth ranked ω_6

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
f_1	ω_3	ω_1	ω_5	ω_4	ω_2	ω_6
f_2	ω_6	ω_1	ω_3	ω_4	ω_5	ω_2
f_3	ω_4	ω_3	ω_6	ω_5	ω_1	ω_2
f_4	$\boxed{\omega_5}$	$\boxed{\omega_3}$	$\boxed{\omega_2}$	$\boxed{\omega_6}$	ω_1	ω_4
f_5	ω_5	ω_1	ω_2	ω_3	ω_6	ω_4
f_6	$\boxed{\omega_6}$	$\boxed{\omega_2}$	ω_5	ω_4	ω_3	ω_1

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
ω_1	f_4	f_3	f_1	f_2	f_5	f_6
ω_2	f_3	f_5	$\boxed{f_6}$	$\boxed{f_4}$	f_1	f_2
ω_3	f_5	f_3	f_6	f_2	f_1	$\boxed{f_4}$
ω_4	f_6	f_4	f_2	f_3	f_1	f_5
ω_5	f_5	f_3	f_2	f_6	f_5	$\boxed{f_4}$
ω_6	f_2	f_3	f_1	$\boxed{f_6}$	f_4	f_5

Step 5: f_4 gives way again to f_6, "proposes" to fifth ranked ω_1

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
f_1	ω_3	ω_1	ω_5	ω_4	ω_2	ω_6
f_2	ω_6	ω_1	ω_3	ω_4	ω_5	ω_2
f_3	ω_4	ω_3	ω_6	ω_5	ω_1	ω_2
f_4	$\boxed{\omega_5}$	$\boxed{\omega_3}$	$\boxed{\omega_2}$	$\boxed{\omega_6}$	ω_1	ω_4
f_5	ω_5	ω_1	ω_2	ω_3	ω_6	ω_4
f_6	$\boxed{\omega_6}$	$\boxed{\omega_2}$	ω_5	ω_4	ω_3	ω_1

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
ω_1	f_4	f_3	f_1	f_2	f_5	f_6
ω_2	f_3	f_5	$\boxed{f_6}$	$\boxed{f_4}$	f_1	f_2
ω_3	f_5	f_3	f_6	f_2	f_1	$\boxed{f_4}$
ω_4	f_6	f_4	f_2	f_3	f_1	f_5
ω_5	f_5	f_3	f_2	f_6	f_5	$\boxed{f_4}$
ω_6	f_2	f_3	f_1	$\boxed{f_6}$	f_4	f_5

Step 6: There are no more "ties." Temporarily held offers becomes realized matches.

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
f_1	$\boxed{\omega_3}$	ω_1	ω_5	ω_4	ω_2	ω_6
f_2	$\boxed{\omega_6}$	ω_1	ω_3	ω_4	ω_5	ω_2
f_3	$\boxed{\omega_4}$	ω_3	ω_6	ω_5	ω_1	ω_2
f_4	$\boxed{\omega_5}$	$\boxed{\omega_3}$	$\boxed{\omega_2}$	$\boxed{\omega_6}$	$\boxed{\omega_1}$	ω_4
f_5	$\boxed{\omega_5}$	ω_1	ω_2	ω_3	ω_6	ω_4
f_6	$\boxed{\omega_6}$	$\boxed{\omega_2}$	ω_5	ω_4	ω_3	ω_1

	1st	*2nd*	*3rd*	*4th*	*5th*	*6th*
ω_1	$\boxed{f_4}$	f_3	f_1	f_2	f_5	f_6
ω_2	f_3	f_5	$\boxed{f_6}$	$\boxed{f_4}$	f_1	f_2
ω_3	f_5	f_3	f_6	f_2	$\boxed{f_1}$	$\boxed{f_4}$
ω_4	f_6	f_4	f_2	$\boxed{f_3}$	f_1	f_5
ω_5	f_5	f_3	f_2	f_6	$\boxed{f_5}$	$\boxed{f_4}$
ω_6	$\boxed{f_2}$	f_3	f_1	$\boxed{f_6}$	f_4	f_5

Figure 6. Ranked Ordered List and Iteration Steps under DA Algorithm (After: Ünver, 2000)

The DA algorithm yields (f_1, ω_3), (f_2, ω_6), (f_3, ω_4), (f_4, ω_1), (f_5, ω_5), (f_6, ω_2) for a consultant-biased logic. Under the intern-biased logic, the resulting matches are slightly different: (f_1, ω_2), (f_2, ω_6), (f_3, ω_4), (f_4, ω_1), (f_5, ω_5), (f_6, ω_3). When an intern ω lists a consultant f in k-th place in her rank order list and the same consultant lists the intern in l-th place, such a (f, ω) match is called a (k ,l) list. Note that (f_2, ω_6) and (f_5, ω_5) pairs have (1, 1) lists, that is, f_2 and ω_6 both select each other as their most preferred partners (the same applies to f_5 and ω_5). Such (1, 1) matches always occur under the DA algorithm.

2. Linear Programming (LP)

Similar to DA algorithm, the LP mechanism used in the United Kingdom takes rank-order lists of interns and consultants as inputs. However, unlike the DA algorithm, these choices are assigned weights denoted by $\alpha_{f,\omega}$. That is, $\alpha_{f\,\omega}$ is the sum of f's weight of ω and ω's weight of f. In the London LP mechanism, choices 1, 2, 3, 4, 5 and 6 are given weights of 36, 28, 21, 15, 10 and 6. Thus (1,1) lists receive the weight $\alpha_{f\,\omega} = 72$, (1, 2) and (2,1) lists each receive the weight $\alpha_{f\,\omega} = 64$, and so forth. The sum of $\alpha_{f\,\omega}$ is then computed for each potential consultant-intern matching pair (f, ω). The resulting weights form the basis for a binary linear programming assignment matching interns to consultants to maximize the value of matches. The problem is described as follows:

$$\text{Max} \sum \alpha_{f\,\omega} \boldsymbol{x}_{f,\omega}$$

$$\begin{aligned} \text{Subject to} \quad & \textstyle\sum \alpha_{f,\omega} = 1 \text{ for all } \omega \\ & \textstyle\sum \alpha_{f,\omega} = 1 \text{ for all } f \\ & \boldsymbol{x}_{f,\omega} \in [0, 1] \text{ for all } f, \omega \end{aligned}$$

$\boldsymbol{x}_{f,\omega} = 1$ denotes a proposed match while $\boldsymbol{x}_{f,\omega} = 0$ means no match between f and ω. The optimal $\boldsymbol{x}$, the matrix of 'proposed matches,' is determined by solving the above LP

problem. The agents (f, ω) who actually listed each other in their rank-order lists and for whom there is a proposed match (i.e., $\mathcal{X}_{f,\omega} = 1$) are matched to each other in the market.

Example:

	1st	2nd	3rd	4th	5th	6th		1st	2nd	3rd	4th	5th	6th
f_1	ω_3	ω_2	ω_1	ω_5	ω_6	ω_4	ω_1	f_3	f_1	f_2	f_4	f_5	f_6
f_2	ω_3	ω_1	ω_2	ω_5	ω_6	ω_4	ω_2	f_3	f_1	f_2	f_5	f_6	f_4
f_3	ω_2	ω_1	ω_3	ω_5	ω_4	ω_6	ω_3	f_2	f_1	f_3	f_4	f_6	f_5
f_4	ω_3	ω_2	ω_1	ω_4	ω_6	ω_5	ω_4	f_2	f_3	f_1	f_6	f_4	f_5
f_5	ω_3	ω_2	ω_1	ω_4	ω_5	ω_6	ω_5	f_1	f_3	f_2	f_6	f_5	f_4
f_6	ω_3	ω_1	ω_2	ω_5	ω_4	ω_6	ω_6	f_3	f_2	f_1	f_6	f_4	f_5
weight (per match per party)	36	28	21	15	10	6		36	28	21	15	10	6

Figure 7. Ranked Ordered List: LP Algorithm (After: Ünver, 2000)

Note that (f_2, w_3) and (f_3, w_2) pairs have (1, 1) lists. The outcome of the London mechanism is given by (f_1, ω_5), (f_2, ω_3), (f_3, ω_1), (f_4, ω_4), (f_5, ω_2), (f_6, ω_6) which provides (4, 1), (1, 1), (2, 1), (4, 5), (2, 4) and (6, 4) lists. This yields a total weight of 276 (= {15 + 36} + {36 + 36} + {28 + 36} + {15 + 10} + {28 + 15} + {6 + 15}). (f_3, ω_2)'s (1 , 1) match is not realized in the solution.

3. Priority

Under this mechanism, each match is assigned a priority in terms of stated preference ranking of consultants and interns. In the Newcastle mechanism, the priority of a (k, l) match is the product of the intern's ranking of the consultant and the consultant's ranking of the intern i.e., k x l. After priorities are assigned, the matches are realized starting from the lowest priority number. The rank ordered lists are summarized below.

Potential Match		ω's position in Q(f)		f's position in Q(ω)		Priority Number
f_1, ω_1	=	3	x	2	=	6
f_1, ω_2	=	2	x	2	=	4
f_1, ω_3	=	1	x	2	=	2
f_1, ω_4	=	6	x	3	=	18
f_1, ω_5	=	4	x	1	=	4
f_1, ω_6	=	5	x	3	=	15
f_2, ω_1	=	2	x	3	=	6
f_2, ω_2	=	3	x	3	=	9
f_2, ω_3	=	1	x	1	=	1
f_2, ω_4	=	6	x	1	=	6
f_2, ω_5	=	4	x	3	=	12
f_2, ω_6	=	5	x	2	=	10
f_3, ω_1	=	2	x	1	=	2
f_3, ω_2	=	1	x	1	=	1
f_3, ω_3	=	3	x	3	=	9
f_3, ω_4	=	5	x	2	=	10
f_3, ω_5	=	4	x	2	=	8
f_3, ω_6	=	6	x	1	=	6

Potential Match		ω's position in Q(f)		f's position in Q(ω)		Priority Number
f_4, ω_1	=	3	x	4	=	12
f_4, ω_2	=	2	x	6	=	12
f_4, ω_3	=	1	x	4	=	4
f_4, ω_4	=	4	x	5	=	20
f_4, ω_5	=	6	x	6	=	36
f_4, ω_6	=	5	x	5	=	25
f_5, ω_1	=	3	x	5	=	15
f_5, ω_2	=	2	x	4	=	8
f_5, ω_3	=	1	x	6	=	6
f_5, ω_4	=	4	x	6	=	24
f_5, ω_5	=	5	x	5	=	25
f_5, ω_6	=	6	x	6	=	36
f_6, ω_1	=	2	x	6	=	12
f_6, ω_2	=	3	x	5	=	15
f_6, ω_3	=	1	x	5	=	5
f_6, ω_4	=	5	x	4	=	20
f_6, ω_5	=	4	x	4	=	16
f_6, ω_6	=	6	x	4	=	24

Figure 8. Priority Algorithm: Score for each potential matches (After: Ünver, 2000)

The ranked ordered lists resulted in the following matches:

Potential Match		ω's position in Q(f)		f's position in Q(ω)		Priority Number
f_2, ω_3	=	1	x	1	=	1
f_3, ω_2	=	1	x	1	=	1
f_1, ω_5	=	4	x	1	=	4
f_5, ω_2	=	2	x	4	=	8
f_6, ω_1	=	2	x	6	=	12
f_4, ω_4	=	4	x	5	=	20

Figure 9. Priority Algorithm Final Matches (After: Ünver, 2000)

B. PAST SIMULATOR DESIGN

In their study to demonstrate the relevance of a two-sided DA matching model, Ng and Soh (2001) constructed their matching simulations using Microsoft Excel and Visual Basic application software. The simulation program was termed the Agent-based Employment Market Simulator (ABEMS).

Two-sided matching aims to match one party to his/her most preferred potential partner. The ranked ordered preference lists compiled by both Sailors and Commands form the basis for the two-sided matching process. The ABEMS was designed to perform three main processes: (a) create a random list of Sailor and Command characteristics through a *Profile generator;* (b) create Sailor/Command preference lists by computing their utility levels based on how they scored their preferred sailor/command in terms of specified preference factors; and (c) create stable matching pairs based on the DA algorithm (using either Sailor or Command biased logic). Summary reports of the simulation runs were then generated. Figure 10 provides a schematic of the components of ABEMS.

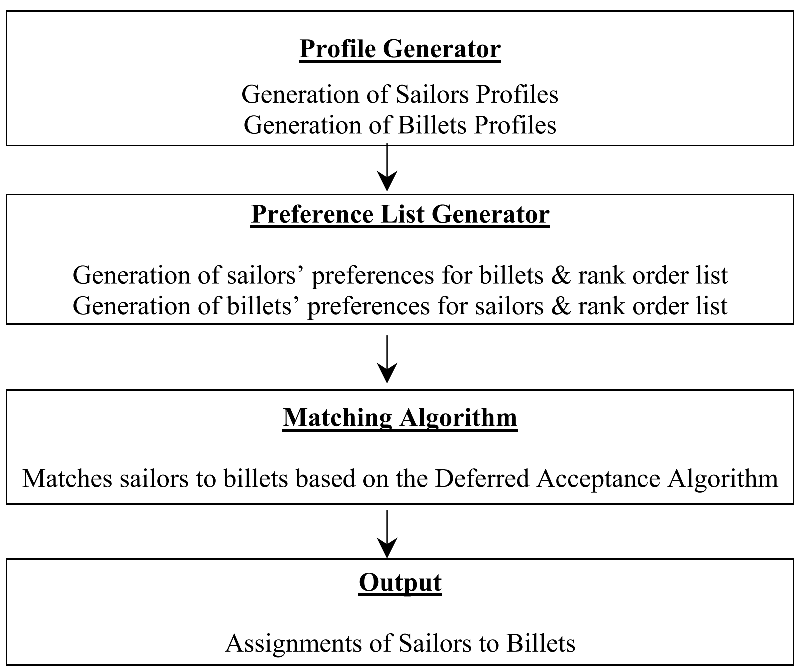

Figure 10. Components of the Agent-Based Employment Market Simulator (ABEMS) (From: Ng and Soh, March 2001)

1. Profile Generator

ABEMS started by mimicking sailor and P1/P2 billets' characteristics as well as the weights both parties place on their preference factors to determine the ranking order of their potential matching "partners." The ABEMS' *Profile generator* module performs this task based on discrete probability distribution specified by the users.

Sailors' and Billets' characteristics - Sailors and Billets are ranked as to how their potential matching "partners" value the characteristics they exhibit. Sailors will score billets based on factors that would satisfy their career/family/individual needs. On the other hand, billets will look for sailors with desired traits that best match their operational requirement. ABEMS assumes three preference factors each for sailor (promotion prospects, billet location, shore billet) and command (sailor's training level, sailor's performance, PCS cost). The *Profile generator* assigns discrete numbers ranging from 1 to 5 to denote aspects of each sailor/command characteristic. Except for promotion prospect of billets, such index values are mere representation of characteristics and are not ordinal value scores. The probability density function of these characteristics are based on close estimation of the existing situation and listed in Table 1.

Table 1. Estimated Probability Distribution Function of Sailor/Billet's Characteristics

	1	2	3	4	5
SAILOR'S CHARACTERISTICS					
Rate	E4 30%	E5 30%	E6 20%	E7 10%	≥E8 10%
Current Billet Location	West 20%	Mid-West 20%	South 20%	N. East 20%	Overseas 20%
Preferred Billet Location	West 20%	Mid-West 20%	South 20%	N. East 20%	Overseas 20%
Training Level	Not trained 10%	Moderate training 30%	Trained 30%	Well-trained 20%	Well-trained w/ experience 10%
Performance	Not promote 10%	Promote 20%	Must promote 40%	-	Early promote 30%

BILLET'S CHARACTERISTICS					
Rating	E4 30%	E5 30%	E6 20%	E7 10%	E8 & E9 10%
Promotion Prospects	Low 10%	Moderate 20%	Average 30%	High 20%	Excellent 20%
Preferred Training Level	Not trained 10%	Moderate training 30%	Trained 30%	Well-trained 20%	Well-trained w/ experience 10%
Billet Location	West 20%	Mid-West 20%	South 20%	N. East 20%	Overseas 20%
Billet-Shore	Sea 20%	-	-	-	Shore 80%

Preference Factors - The value on each preference factors is derived based on the degree to which a sailor's current characteristics match his/her potential billet's desired sailor's characteristics (and vice versa). The better the characteristics match, the higher the score. This is translated into a separate index of 1 to 5, with 5 being the highest value for a preference factor in the utility function. The computation of these indices is covered in greater detail in the next section on "Preference list generator."

Weights on Preference Factors - ABEMS assumes a Cobb-Douglas utility function to determine the total value that sailors/commands derive based on the various preference factors and these factors' relative weights. The weight placed by the *i*-th sailor on the *j*-th billet characteristic is α_{ij} (while β_{ji} denotes the weights the *j*-th billet places on the *i*-th sailor characteristic). These weights (α_{ij} and β_{ji}) are randomly derived through the random number generator function (RAND( )[1]) in MS Excel.

2. Preference List Generator

Preference lists are generated for the sailor over each billet and for the billet over each sailor based on the individual preferences of the sailor and the billet. The preferences are modeled by a Cobb Douglas (multiplicative) utility function. The Cobb Douglas utility function was chosen as it allows for: (1) diminishing marginal rates of return over the

1 RAND( ) returns an evenly distributed value between 0 & 1 (inclusive).

individual preference factors, and (2) trade-offs between the preference factors depending on individual tastes and incorporated through the weights assigned to each factor. However, the Cobb Douglas utility function assumes that the factors are interdependent; that is consuming more of one factor will increase the value received from the other factors.

a. Sailor Preference Lists

In generating sailor preferences over each billet, the preference factors that are deemed important to the sailor include the billet's promotion prospects, the fit between the sailor's preferred location and the billet location as well as whether the billet is a shore or a sea billet. The sailors' total utility for the job is then modeled by the following utility function:

$$U_s = BPI^{\alpha(P)} * L^{\alpha(SPL)} * BS^{\alpha(BS)}$$

Where:

U_s = Total utility of the sailor for a particular billet

$BPI^{\alpha(P)}$ = Utility derived from promotion prospects of the billet

$L^{\alpha(SPL)}$ = Utility derived from fit between sailor's preferred location and billet location (SPL & BL)

$BS^{\alpha(BS)}$ = Utility derived from getting a shore billet

$\alpha(P) + \alpha(SPL) + \alpha(BS) = 1$

The weight (α) for each preference factor is generated randomly but the sum of the weights must equal 1.

Billet Promotion Index – The Billet Promotion Index captures the satisfaction that sailors place on getting a high profile and more challenging job. The higher the index, the more satisfaction the sailor will derive from the job as it provides a higher prospect for

promotion. A score of 5 is given to a job that has high promotion prospects and a score of 1 is given to a job that has low promotion prospects.

Location Fit – The Location Fit factor captures the fit between the billet's location and sailor's preferred location. A score of 5 indicates that there is a fit between the sailor's preference and the billet location whilst a score of 1 indicates that there is no fit between the sailor's preference and the billet location.

Billet Shore – The Billet Shore factor captures the sailor's satisfaction from getting a shore billet. The sailor will derive greater satisfaction from a shore billet and is assigned a score of 5 whilst the sea billet will be assigned a value of 1, corresponding to a lower satisfaction level.

A summary of the scores that are assigned to each preference factor is listed in Table 2.

Table 2. Pay-off Matrix for the Preference Factors in the Sailor Utility Function

Billet Promotion Index (BPI)	Score	Location Fit (L = SL-BL)	Score	Billet Sea or Shore (BS)	Score
Excellent	5	SL = BL	5	Shore	5
High	4	SL ≠ BL	1	Sea	1
Average	3				
Moderate	2				
Low	1				

Before generating the sailor preference lists, the simulator will only allow the sailor to apply for jobs within the sailor's rate, that is SR and the BR must be the same. The simulator then generates the Sailor's Preference List by calculating the Sailor Utility (U_S) for each eligible billet and ranks the billets in decreasing order of utility.

b. Command Preference Lists

The preference factors that are deemed important to the command include the training level attained by the sailor, the quality of the sailor as determined by the likelihood of promotion to the next rank, and the PCS costs associated with moving the sailor to the job. The command's total utility for the job is then modeled by the following utility function:

$$U_c = TL^{\beta(TL)} * SPI^{\beta(SPI)} * PCS^{\beta(PCS)}$$

Where:

U_c = Total utility of the command for a particular sailor

$TL^{\beta(TL)}$ = Utility derived from training level of the sailor

$SPI^{\beta(SPI)}$ = Utility derived from the performance of the sailor

$PCS^{\beta(PCS)}$ = Utility derived from getting lower PCS costs

$\beta(TL) + \beta(SPI) + \beta(PCS) = 1$

The weight (β) for each preference factor is generated randomly but the sum of the weights must equal 1.

Training Level – The training level captures the difference between the command's desired training level (BTL) and the sailor's actual training level (STL). The difference in training level becomes a training gap and a larger training gap is assigned a lower score; a score of 5 is given when there is no training gap and a score of 1 is given when there is a very larger training gap.

Sailor's Performance Index – The sailor's performance index captures the command's satisfaction in being assigned a sailor who has a higher performance rating.

Sailors in the early promote zone are assigned a score of 5 whilst those in the not promote zone are assigned a score of 1.

PCS Cost – The PCS factor captures the value that the command places on having a low PCS cost associated with the assigned sailor. Command's that want to minimize PCS Costs will prefer a sailor whose current billet is located nearer the command. A cost matrix was developed and a cost score of 1 to 5 is assigned to the relative costs for moving from one location to another. A score of 5 indicates a very low cost whilst a score of 1 indicates a very high cost.

The scores assigned to the training level and sailor's promotion index are given by the payoff matrix in Table 3, whilst the score for the PCS Cost factor are given in Table 4.

Table 3. Pay-off Matrix for Training Level (TL) and Sailor's Promotion Index (SPI) Preference Factors in the Command Utility Function

Training Level ($\lvert$BTL-STL$\rvert$)	Score	Sailor's Performance Index (SPI)	Score
No Training Gap	5	Early Promote	5
Small Training Gap	4	Must Promote	3
Moderate Training Gap	3	Promote	2
Large Training Gap	2	Not Promote	1
Very Large Training Gap	1		

Table 4. Pay-off Matrix for PCS Cost Preference Factor in the Command Utility Function (5 = Very Low Cost, 4 = Low Cost, 3 = Moderate Cost, 2 = High Cost, and 1 = Very High Cost)

Location	W	MW	S	NE	Overseas
W	5	4	3	2	1
MW	4	5	4	3	2
S	3	4	5	4	3
NE	2	3	4	5	4
Overseas	1	2	3	4	5

Using the values in the pay-off matrices, the simulator then generates the Command's Preference List by calculating the Command Utility (U_c) for each eligible sailor and ranks the sailors in decreasing order of utility.

3. Matching Algorithm

The ABEMS adopts the Deferred Acceptance (DA) mechanism described in the opening section of this chapter to create stable matching pairs. The DA algorithm is based on either sailor-biased or command-biased logic.

a. Sailor Biased Algorithm

The DA matching logic can be achieved by placing sailors' priority over commands'. This results in a sailor-biased match, which indicates that sailors who are matched cannot possibly find another mutually beneficial match with a billet ranked higher in their preference list. The commands however, will only be matched to the sailor that is ranked lowest in its preference list but still represents a stable match. In this matching algorithm, the utility of the sailors is maximized and utility of the commands is minimized, while still ensuring a stable match. The match will be the optimal stable for the sailors as a group.

b. Command Biased Algorithm

The DA assignment could also be achieved by placing the command's priority over the Sailor's. This will result in a command-biased match, reflecting the current detailing process where detailers try to assign sailors to prioritized jobs while considering the sailor's preferences. Similar to the sailor-biased matching algorithm, the commands' utilities are maximized while still ensuring a stable match. The match will be the optimal stable match for the commands as a group.

C. STRENGTH & WEAKNESS OF DEFERRED ACCEPTANCE (DA) AND LINEAR PROGRAMMING (LP) ALGORITHMS

This section discusses the trade off between DA and LP mechanisms. The priority mechanism is no longer in use because of problems that led to hospitals hiring interns up to two years in advance of their graduation dates. Two primary issues that decide if an algorithm is desirable include the questions of: (a) whether stable matchings can be achieved and (b) whether the welfare of the matching parties is maximized. The following explains the notion of stable matching and welfare maximization. The trade-off is that DA ensures stable matching but does not guarantee welfare maximization; the LP algorithm ensures welfare maximization but does not guarantee stable matching.

1. Stability of Matching Algorithm

A matching or assignment is said to be "stable if it is not blocked by any individual or any pair of agents" (Roth, 1990). In our case, this means that there is no instance where both a particular command and a particular sailor would prefer each other above their assigned matches. For example, consider the following match where Sailor 1 is assigned to Command A and Sailor 2 is assigned to Command B:

Match:	Command A	Command B
	Sailor 1	Sailor 2

If Sailor 1 prefers to be assigned to Command B rather than Command A and Command B prefers to be assigned Sailor 1 rather than Sailor 2, a blocking pair is said to exist and the match is not considered stable; both Sailor 1 and Command B have an incentive to block the matching process and seek a match with each other. Similarly, if Sailor 2 would rather not be assigned for this requisition cycle than go to Command A, Sailor 2 will individually block the match and the match is regarded as unstable.

Stable matching upholds the integrity of the mechanism, as otherwise the two matching parties would seek a preferred partner outside of the matching system. Both Gale and Shapley (1962) and Roth (1990) have shown that the DA algorithm described in Section A will ensure a stable match. Stability is considered a strength as experience in the centralized matching markets for physicians in the United Kingdom have shown that achieving stability is important to ensuring the success of the matching process (Roth, 1991). Failure of matching systems has been attributed to unstable matches resulting in an increasing number of physicians and hospitals choosing to negotiate outside of the central matching system.

Stable matchings are not guaranteed under LP mechanisms. Note that (1,1) lists are not always realized in the LP mechanism. From the example in figure 7, the pair (f_3, ω_1) could be blocked by the (1, 1) match of (f_3, ω_2).

Priority mechanisms can be unstable. That is, there can be a consultant-intern pair who each prefers the other rather than their match. However, a (1,1) match is always realized in this particular mechanism. Priority mechanisms were abandoned after several years of trial in the field. In Newcastle, 80% of the lists consisted of a single choice. This was evidence of early agreements. Similar centralized matching procedures failed and could not fix the unraveling problem that first appeared during the decentralized matching era.

2. Optimality of Matching Algorithm

In the sailor biased algorithm, there are no other stable matches in which a sailor will get a match that is better than the sailor-biased match whilst no command will get a match that is worse than the current match. Conversely, in the command biased algorithm, there are no other stable matches in which a command will get a match that is better than the command-biased match whilst no sailor will get a match that is worse than the current match. In practice, there was little difference between the two solutions (sailor or command biased); usually just 2 pairs reversed out of some 20,000 matches according to a study by

Roth (1999). The sailor/command biased logic creates a situation where either the Sailor or the Command will fulfill their preferences at the expense of the other. This is a weakness of DA algorithm, as assignments completed under a more equitable criterion of optimality could improve the welfare of the system as a whole, as compared to either a sailor or command optimal criterion. The primary aim of the LP algorithm was to maximize welfare without any bias to either party. Nevertheless, this strength of LP algorithm needs to be weighed against its inability to ensure stable matching.

D. POTENTIAL ENHANCEMENTS TO ABEMS

ABEMS is useful in generating the set of sailor and command preferences and assigning matches based on a rank ordered list of these preferences using the Deferred Acceptance algorithm. However, because of the manner in which it generates the preferences and the matching algorithm that is used, there is room for further enhancements. Specifically, these involve: (1) Generation of Preferences, (2) the Utility Function; (3) Rules in Shortlisting and (4) Empirical comparison of Algorithms.

1. Generation of Preference Factors

The factors that the sailor uses to choose between billets are postulated to be the promotion prospects from going to that billet, the location of that billet, and whether or not the billet is a shore billet. On the other hand, the factors that the command uses to choose between sailors include the extent to which the sailor meets the command's training requirements, the sailor's prior performance and the PCS costs associated with moving the sailor. While these factors appear to be reasonable estimates of sailors/billets' preference factors, they were mere theoretical assumptions and were not validated by rigorous survey results. An investigation has recently been conducted on the Aviation Support Equipment Technician (AS) rating by Butler and Molina (2002) to determine the actual preference factors for this rating. A more realistic generation of the preference factors may thus be obtained by incorporating their findings.

2. Utility Function

The ABEMS assumes a Cobb Douglas utility function. While the Cobb Douglas utility function reflects two typical economic properties (diminishing marginal rates of return and interdependencies between the preference factors), this may not represent sailor/billet utility functions. For example, the preference factors of billet promotion prospects and its location need not be interrelated. Such a "substitute" characteristics also reduces the absolute utility score, compresses the range of utility and further underestimates the strength of an LP algorithm. An alternative utility function, either totally additive or partial multiplicative/additive should be considered for simulation study.

3. Rules in Short-listing

The ABEMS is stringent in ensuring that fully qualified sailors are placed on the shortlist for matching purposes. However, the existing rule defines a sailor as being "qualified" only if his/her ratings are exactly the same as the billets. In reality, it is common for sailors to one-up, that is, hold a billet position that has a higher rating than their held rank. The existing stringent short-listing rule unnecessarily truncates the list of eligible billets and makes it much more difficult to achieve successful matching. We should consider billets that are of higher or lower rate than the sailor's held rank. The penalty for any non-exact rate match would be inherently present through a minimum score for the preference factor. The deviation would also be limited to 1 rate.

4. Empirical Comparison of Algorithms

Ng and Soh's simulation study (2001) primarily investigated the following: (1) the optimal intervals between matching, using constant preference list lengths of 5; (2) the optimal preference list length (holding number of sailors and billets constant); (3) the optimal preference length for increasing intervals in the requisition cycles; (4) the effect of increasing proportions of P1 billets in the matching process; (5) the effects of sailor biased and command biased matching; and (6) the optimal possible matching outcome for a 2-week and 8-week sample using different preference lengths.

The ABEMS used the DA algorithm because of the presumed high value placed on stable matching qualities versus other advantages of utility maximization. It did not provide empirical evidence to allow a vigorous comparison of the net benefits between DA and LP algorithms. This thesis will seek to enhance ABEMS so that results of the LP algorithm can be explored and compared to the DA mechanism.

5. Rank Order 'Tie Breaker'

It is not uncommon for sailors and commands to derive the exact same utility level from more than one command and sailor, respectively, in their preference list. This may be due to the narrow range of utility scores inherent in the multiplicative Cobb Douglas model. A tie breaker is therefore necessary to derive the rank order. The ABEMS adopts a simple 'first-come-first-serve' basis to break the tie. That is, for any sailors/commands that offer the same utility to any command/sailor, whichever sailor/command is evaluated first will be ranked higher. This is arbitrary. A better tie-breaker may be to fall back on the utility score of the preference factors derived by the sailors/commands. For example, if sailor *i* scores both command *m* and *n* equally in terms of total utility, it could rank command n higher if the latter provides greater utility for the most valued preference factor (e.g., promotion prospects) relative to the other preference factors of location and sea shore rotation.

E. AN ALTERNATIVE

An alternative to ABEMS would be to construct a simulator that incorporates the actual sailor/command preference factors, an alternative utility function, a less stringent short-listing rule and uses a Linear Programming (LP) matching logic.

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IV. THE NAVY ENLISTED DISTRIBUTION SIMULATOR (NEDSIM)

A. OBJECTIVE OF REVISED SIMULATION EXERCISE

The primary objective of this revised simulation exercise is to investigate the relative strengths of Deferred Acceptance (DA) and Linear Programming (LP) algorithms. The question is to see which of these methods has a net overall advantage to be subsequently deployed in the revised Navy Enlisted Distribution and Assignment system. The revised simulator will build upon prior simulation software (ABEMS) by Ng and Tan (2001). Relevant performance indicators (to be covered in the next section) will be derived to measure the main benefit and cost of the respective methods.

After results were obtained from a reasonable number of simulation runs (say 100 or about four year's equivalent based on a two week requisition cycle), the difference in the performance indicators from both algorithms will be compared using statistical analysis to ascertain their significance.

B. PERFORMANCE MEASURES

Tangible performance statistics are required to compare the net benefits or shortcomings between the LP and the DA algorithms. The performance indicators were formulated to measure both the quantity as well as the quality of the matches obtained via both methods.

1. Quantity of Matches

The main aim of the matching process is to make as many matches as possible. To determine the extent to which this aim is met, the percentage of matches is obtained for the sailor as well as for the billet. The percentage of matches is defined as:

Percent Sailor Match = No. of Matches / Total no. of Sailors available for Matching

Percent Billet Match = No. of Matches / Total no. of Billets available for Matching

2. Quality of Matches

Besides the quantity of matches, the quality of matches obtained is also important. Two main measures were formulated to determine the quality of the matches in terms of utility of the matches as well as the percentage of unstable matches. The specific measures are the total utility per match, sailor utility per match, command utility per match and the percentage of unstable matches. The measures are defined as:

Average Total Utility = Total Sailor & Command Utility / No. of Matches

Average Sailor Utility = Total Sailor Utility / No. of Matches

Average Command Utility = Total Command Utility / No. of Matches

Percent Unstable Matches = No. of Unstable Matches / No. of Matches

3. Composite Performance Measure

A composite performance measure was developed to incorporate both the quality and the quantity of the matches. This will provide a single measure to compare between the two algorithms. The composite performance measure is defined as:

$$CPM = \text{Average Total Utility}^{w(u)} * \text{Percent Stable Matches}^{w(SM)} * \text{Percent Matched}^{w(M)}$$

where *w(U) is the weight assigned to Average Total Utility*

w(SM) is the weight assigned to Stable Matching

w(M) is the weigh assigned to Successful Matches

For this study, it is assumed that each performance factor is weighted equally, but this could be adjusted in future research. For example, if there is relatively higher cost from unstable matching, the weightage w_{SM} could be raised accordingly.

C. DESIGN OF NEDSIM

Prior simulation software (ABEMS) by Ng and Soh (2001) is powerful in generating matching results under the DA algorithm. The sub-modules that generate individual sailor and command characteristics as well as the preference lists are also useful. This study still requires similar preference list generation logic and the matching results from the DA algorithm. It is therefore more expedient to adapt ABEMS for these functions into the revised simulator. Additional capability to incorporate the Linear Programming algorithm and other content changes were necessary to form the final modules; in a model called the Navy Enlisted Distribution Simulator (NEDSim) to distinguish it from ABEMS. NEDSim also incorporates a few new functions to make the simulation more realistic and complete. These new functions include: (1) more realistic sailor and command preference factors, (2) an alternative utility function, (3) the LP matching algorithm and (4) checks for unstable matching.

1. Preference Factors

Butler and Molina (2002) conducted surveys and focus groups on the Aviation Support Equipment Technician (AS) rating to determine why sailors prefer a particular billet and why a command prefers a particular sailor for a specific billet. The results of the survey are incorporated in NEDSim.

a. Sailor Preference Factors

Sailors were asked to answer a questionnaire to indicate preference factors that were important to them when considering which billet to request. The sailor preference factors were categorized into five main groups, consisting of job factors, location factors, family life factors, incentive factors and training and education factors. Of the five factors, family life, location and job factors were found to be the three most important factors, with

more than 30 percent of respondents saying that they were important. The three factors of family life, location and job are therefore incorporated into NEDSim. Figure 11 illustrates the most important sailor preferences by percentage of respondents.

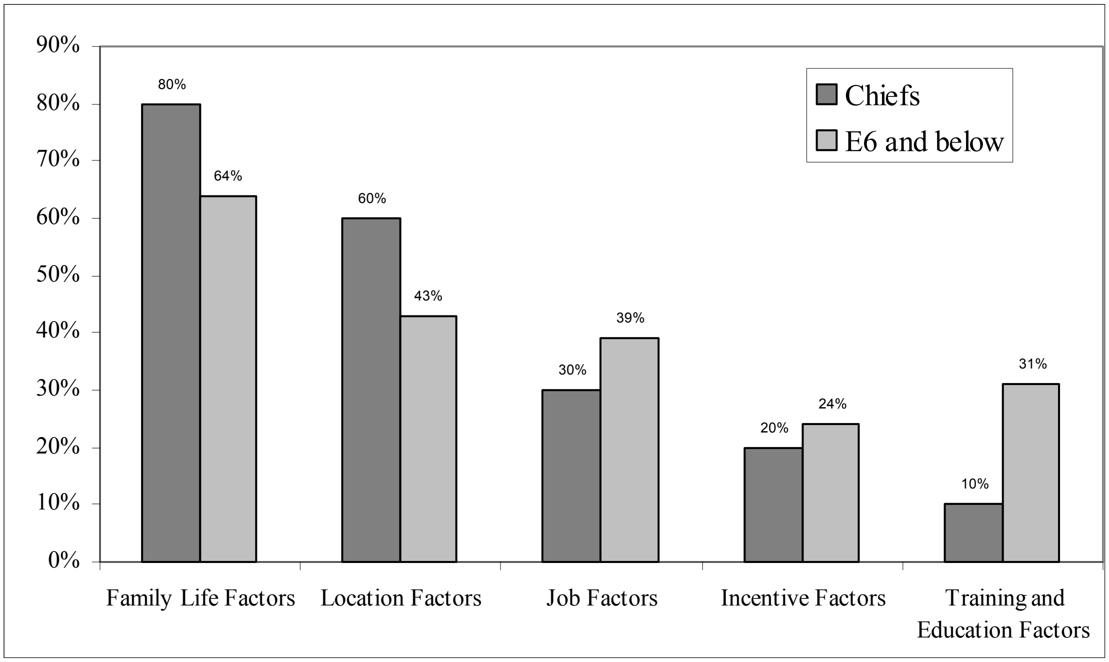


Figure 11. Most Important Sailor Preferences by Percentage of Respondents (N=100) (After: Butler and Molina, March 2002)

Family Life Factor – Family life was the most important preference factor, with 80 percent of the chiefs and 72 percent of the E-6 and below rates saying family life was important. Of the 11 family life factors, the three attributes that more than 50 percent of respondents cited as important include: (1) whether the civilian spouse can find a job (90 percent of chiefs and 72 percent of E-6 and below rates), (2) co-location with the military spouse for sailors who were married to another active duty member (70 percent of chiefs and 42 percent of E-6 and below rates) and (3) the opportunity for family to accompany them for those who were not married to another active duty military member (60 percent of chiefs and 55 percent of E-6 and below rates).

Location Factor – Location was cited as the next most important sailor preference factor, with 60 percent of the chiefs and 43 percent of those E-6 and below rates saying that they were important. Of the 10 location attributes, the three most important

attributes cited by more than 50 percent of the respondents as important, include: (1) affordable cost of living (70 percent of chiefs and 62 percent of E-6 and below rates), (2) affordability of a house (50 percent of chiefs and 62 percent of E-6 and below rates) and (3) easy transition to civilian life (50 percent of chiefs and 25 percent of E-6 and below rates).

Job Factor – Job attributes were cited as the third most important sailor preference factor, with 30 percent of the chiefs and 40 percent of those E6 and below saying that they were important. Of the 10 job attributes, the two attributes cited by more than 50 percent of the respondents as important, include: (1) job's ability to help advancement (80 percent of chiefs and 85 percent of E-6 and below rates) and (2) shore duty (60 percent of chiefs and 60 percent of E-6 and below rates).

The sailor preference attributes that will be used in NEDSim are those that more than 50 percent of respondents said were important; they are listed in Figure 12.

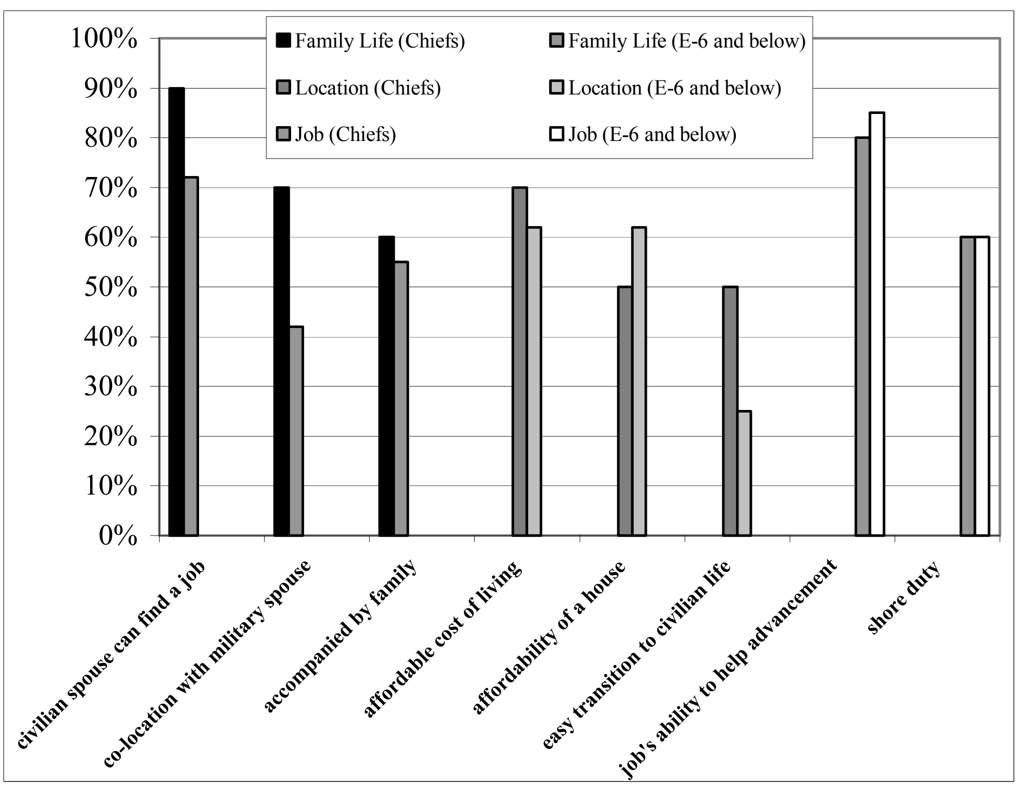


Figure 12. Most Important Sailor Preference Attributes incorporated in NEDSim by Percentage of Respondents (N=100) (After: Butler and Molina, March 2002)

b. Command Preference Factors

Commands were asked to answer questions in the survey to rank six preference attributes for assigned sailors. The six preference factors included whether the sailor: (1) had the correct paygrade, (2) was trained enroute, (3) had the specific NEC, (4) had specific experience for the job, (5) was promotable, and (6) whether there would be a gap in assignment. It was found that the top three command preference factors, with more than 50 percent of respondents saying that they were important, included sailors having the correct NEC (70 percent), the correct paygrade (67 percent) and no billet gap (56 percent). However, due to the small return sample size of 26, all the command preference factors will be incorporated in NEDSim except for two: being trained enroute, as the sailors have to be trained before they take up the billet, and whether there would be a gap in assignment, since this is not a characteristic of the sailor over which the command can exercise a choice. Figure 13 shows the most important command preference factors.

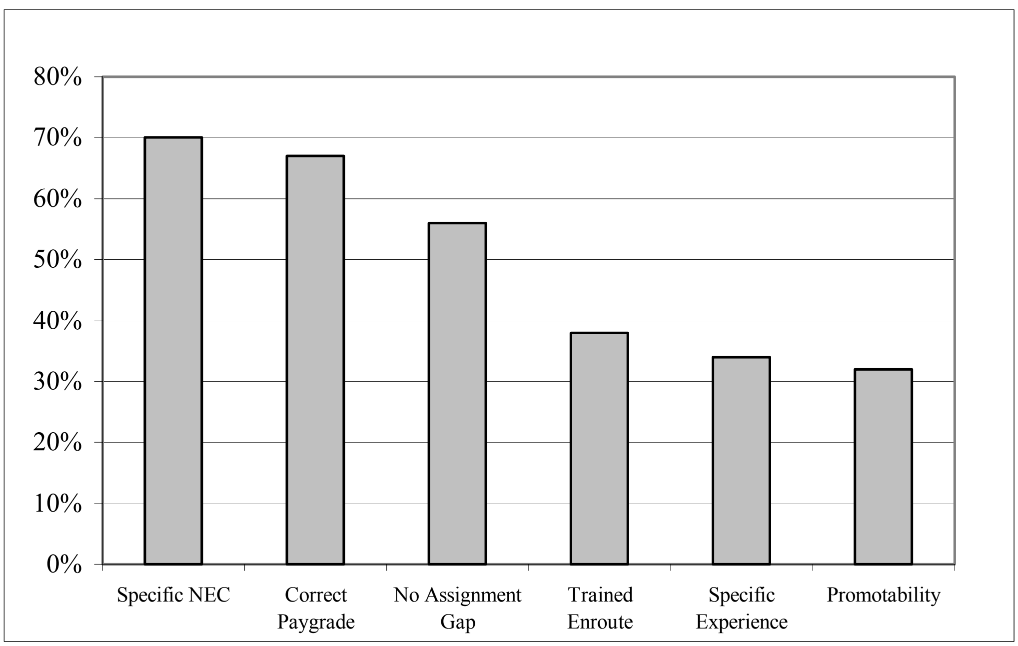


Figure 13. Most Important Command Preference Factors by Percentage of Respondents (N=26) (After: Butler and Molina, 2002)

2. Utility Function

Two general utility functions can be used for the purposes of simulation: the multiplicative form and the additive form. The multiplicative form (Cobb Douglas) allows for diminishing marginal rates of return for the individual factors and interrelationships between the individual factors subject to a total utility constraint. These interrelationships assume that the individual factors are affect the value derived from the other factors (e.g., a more preferred location factor increases the value of the visibility factors), but this may not be the case for job preference factors. In fact, job preferences may be independent, since having more of one preference factor does not influence the individual's utility levels of the other preference factors. Hence, for simulation of billet selection choices, NEDSim uses the additive utility function with the general form:

$$U = \alpha_1 A + \alpha_2 B + \alpha_3 C$$

where:

U	=	Total utility of the sailor/billet for a particular billet/sailor
A, B, C	=	Utility derived from factor A, B or C.

$\alpha_1 + \alpha_2 + \alpha_3 = 1$

3. LP Matching Algorithm

a. Utility Maximization

The general LP model for the assignment problem has been described in section A.2 of Chapter 3. The preference lists for both sailors and billets are the same as those generated for DA matching. However, the ranked-order within the preference list has much greater significance for the DA algorithm than the LP method. The aim of the LP model is to maximize the combined utility of both sailors and billets that are successfully matched (The DA method emphasizes only one party's utility depending on whether it is

Sailor or Command-biased). Thus, the objective function for the LP model is the sum product of the combined utility for any unique matching pair and the binary value of 0 or 1 (indicating if a match occurs). In the LP example from Chapter 3, a pre-determined value is assigned to the matching pairs in terms of the original ranked position of sailor/billet in their matching partners' preference list. For this study, the relative value of the matching pairs is already inherent in their combined utility. The constraints reflect the condition that each sailor/billet can only be matched to one billet/sailor.

B. Stability of Matching

Unlike the DA algorithm, where its benefits and shortcomings are reflected in utility level and percentage of matching pairs, the LP method has an additional measurement in terms of blocking pairs or unstable matching. Because the LP method does not guarantee 100% stable matching, the simulator will go through all matching pairs from the LP utility maximization and investigate for any blocking pairs.

D. COMPONENTS OF NEDSIM

NEDSim uses two-sided matching to match one party to his/her most preferred potential partner. The ranked ordered preference lists compiled by both Sailors and Commands form the basis for the two-sided matching process. NEDSim was designed to perform three main processes: (a) create a random list of Sailor and Command characteristics through a *Profile generator;* (b) create Sailor/Command preference lists by computing utility levels based on sailor/command scores over specified preference factors with an additive utility function; and (c) create matching pairs based on both the DA and LP matching algorithms. Summary reports of the simulation runs are also generated. Figure 14 provides a schematic of NEDSim's components.

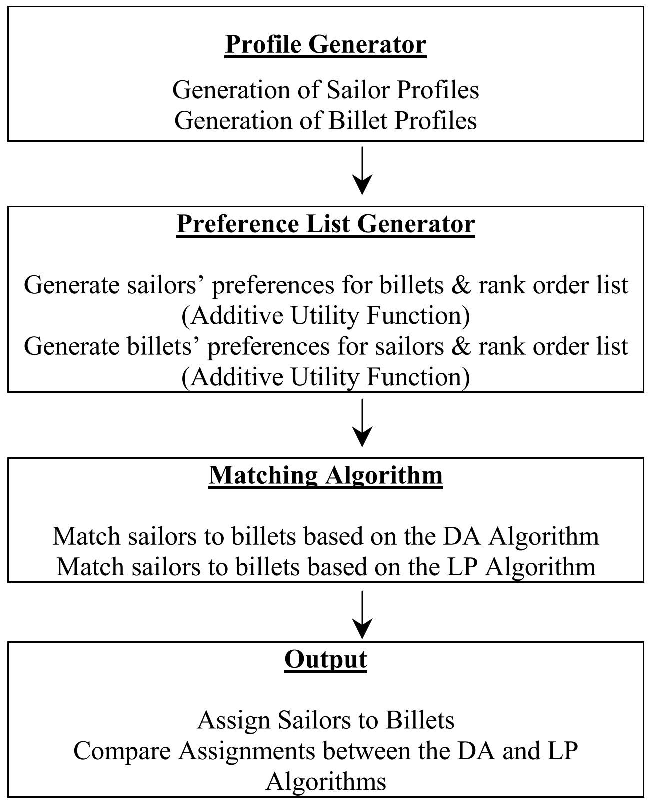


Figure 14. Components of the Navy Enlisted Distribution Simulator (NEDSim)

1. Profile Generator

NEDSim generates the billets' characteristics as well as the weights that sailors and commands place on their preference factors to determine the rank order of their potential matching "partners." NEDSim's *Profile generator* module performs this task based on discrete probability distributions determined through characteristics of the community used for the simulation.

Sailors and Billets are ranked as to how their potential matching "partners" value the characteristics they exhibit. Sailors will score billets based on factors that would satisfy their career, family or individual needs. On the other hand, billets will look for sailors with desired traits that best match their operational requirements. NEDSim assumes three preference factors for the sailor (family life, location and job) and four preference factors for

the command (NEC, paygrade, experience and sailor promotability). The *Profile generator* generates the profile of sailors and commands based on the AS community characteristics.

a. Sailor Characteristics

NEDSim profiles sailors according to their grade, NEC, experience and performance. The probability distribution of the sailor's characteristics is listed in Table 5 and is drawn from the Enlisted Master File (EMF) for the AS rating.

Table 5. Probability Distribution of Sailor Characteristics

Characteristic	**Probability Distribution**				
Rate (Paygrade)	E3 10%	E4 30%	E5 31%	E6 21%	≥E7 8%
NEC	7600 32%	7607 24%	7612 11%	7614 18%	7699 15%
Experience	CV 30%	LPD / LHA / LHD / MCS 14%	Other Sea 12%	AIMD 29%	Other Shore 15%
Performance	Not promote 10%	Progressing 30%	Promotable 30%	Must promote 20%	Early promote 10%

Rate of Sailor – The rate profile of the sailor is determined from the Enlisted Master File for the AS community.

NEC of Sailor – Similar Navy Enlisted Classifications (NEC) have been grouped into five categories of NECs for ease of computation. The NEC distribution is also obtained from the EMF.

Table 6. NECs under each NEC Group for NEDSim

NEC Group	NECs
7600	7222, 8364, 8880
7607	7601, 7603, 7606, 7607
7612	7610, 7612, 7616, 7617
7614	7614, 7618
7699	7699

Experience of Sailor – The AS rated sailor comes from several sea platforms and shore establishments. The sea platforms include aircraft carriers (CVs), amphibious ships (LHDs & LHAs) and mine command ships (MCS). Air squadron and OCONUS assignments are considered as fleet assignments and are classified under the other sea category. Shore assignments include billets in AIMD detachments, recruiting commands, training commands and air wings. A breakdown of these billets is given below.

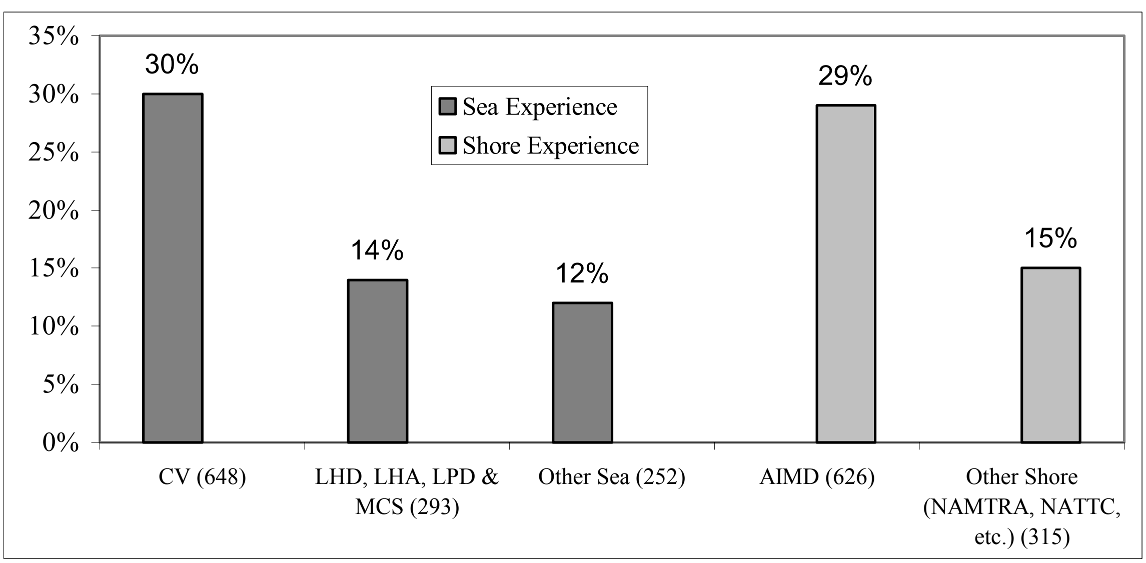


Figure 15. Breakdown of Sailor Experience by Type of Platform / Establishment

Performance of Sailor – The performance of the sailor is assumed to be normal through the five categories of not promote, progressing, promotable, must promote and early promote.

b. Billet Characteristics

NEDSim profiles billets according to their rate, location, promotion prospects of the billet and whether or not the billet is ashore. The billet characteristics are listed in Table 7.

Table 7. Probability Distribution of Billet Characteristics

Characteristic	**Probability Distribution**				
Rate (Paygrade)	E3 14%	E4 28%	E5 32%	E6 18%	≥E7 8%
Location	East Coast (CEC) 33%	Gulf Coast (CGC) 13%	South West (CSW) 25%	North West (CNW) 10%	OCONUS (OPL) 19%
NEC	7600 24%	7607 22%	7612 16%	7614 26%	7699 12%
Visibility	Low 16%	Moderate 20%	Average 24%	High 21%	Excellent 19%
Platform Profile	CV 28%	LPD / LHA / LHD / MCS 13%	Other Sea 8%	AIMD 40%	Other Shore 9%
Shore	Sea 51%			Shore 49%	

Billet Rate - The billet rate profile is obtained from the billet master file for the AS rating.

Location of Billet – The billets are grouped according to the different Navy regions of Continental U.S., East Coast (CEC), Continental U.S., Gulf Coast (CGC), Continental U.S., Southwest (CSW), Continental U.S., Northwest (CNW) and Outside the Continental U.S., Pacific and Atlantic (OPL). The following table lists down the proportion of people in each of the regions for the AS community.

Table 8. Distribution of the AS rated Billets across the different Navy Regions

Navy Region	No. of Billets	Proportion (%)
Continental U.S., East Coast (CEC)	711	33%
Continental U.S., Gulf Coast (CGC)	281	13%
Continental U.S., Southwest (CSW)	548	25%
Continental U.S., Northwest (CNW)	218	10%
Outside the Continental U.S., Pacific and Atlantic (OPL)	407	19%
	2165	100 %

NEC – The billet NEC profile is obtained from the Billet Master File for the AS rated sailors.

Visibility of the Billet – The visibility of the billet is an indication of how the billet can enhance the promotion prospects of the sailor because of the profile that is accorded to the sailor. The visibility is approximated by how much the billet rate is above the sailor rate. If the job requires skills that are above his current rate, the visibility of the billet is higher.

Billet Shore and Platform – The number of sea to shore billets for the AS rating stands at 51% to 49%. A breakdown of the sea and shore billets is given in the following figure.

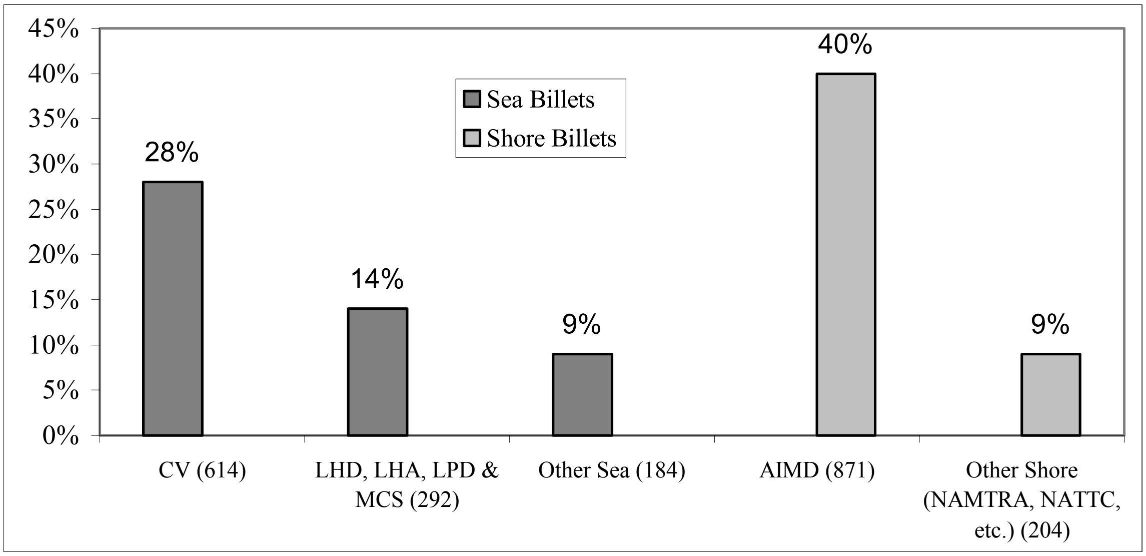


Figure 16. Breakdown of Sea and Shore Billets

2. Preference List Generator

Preference lists are generated for the sailor over each billet and for the billet over each sailor, based on the individual preferences of the sailor and the billet. The preferences are modeled by an additive utility function, as the individual preference factors are independent and not interrelated with each other. Weights for the preference factors are generated randomly through the random number generator function (RAND()[2]) in MS Excel. The preference factor weights for the sailors are denoted by α_{factor} and the preference factor weights for the commands are denoted by β_{factor}.

NEDSim will activate the simulator specifications dialog box when the user clicks the "*start simulation"* command button on the main worksheet. The dialog box will allow the user to specify various parameters for the simulation, including the number of sailors to be matched, the number of priority 1 requisition billets, the number of priority 2/3 requisition billets, and the preference list length. Each field of entries and the command button within the Start Simulation dialog box will need to be entered or activated to complete one successful run. Users can proceed with one simulation run by pressing the

2 RAND() returns an evenly distributed value between 0 (inclusive) & 1.

seven steps from "regenerate random numbers" to "command biased matching". Alternatively, the command button "automate simulation Step(1-7) for 10 runs" allows user to do so automatically for 10 simultaneous runs.

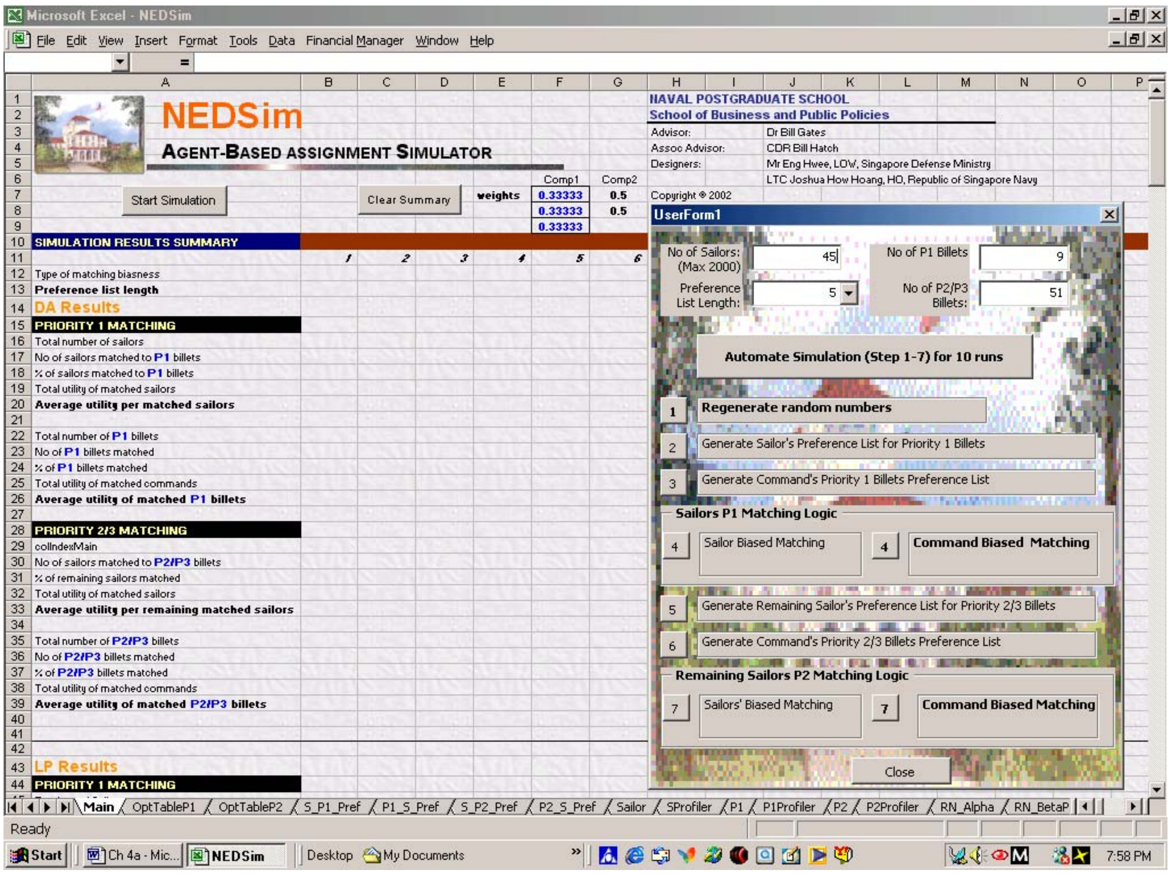


Figure 17. Main Page of NEDSim

a. *Sailor Preference Lists*

In generating the sailors' preferences over each billet, the preference factors that are deemed important to the sailor include family life, location, and job factors. The sailors' total utility for the job is then modeled by the following utility function:

$$\text{Sailor Utility} = \alpha_{FL}\,(\text{Family Life}) + \alpha_{L}\,(\text{Location}) + \alpha_{J1}\,(\text{Promotion}) + \alpha_{J2}\,(\text{Shore})$$

$$\text{where}: \alpha_{FL} + \alpha_{L} + \alpha_{J1} + \alpha_{J2} = 1$$

(each α is generated randomly)

Family Life Preference Factor – The Family Life Factor is determined primarily by whether the billet location favors the civilian spouse's employment opportunities. This factor is captured by the recent job growth rates at the billet location, as higher recent job growth rates imply that the spouse will more likely be employed. The billet location with higher recent job growth rates will score higher in the Family Life factor, and the higher the score the more satisfaction the sailor will derive from the factor. A score of 5 is given to a job that has excellent family life qualities and a score of 1 is given to a job that has low family life qualities. As the billet regions comprise many cities, representative cites were used for the billet locations. OCONUS locations were the most diverse as it spanned many world regions and it was assumed that spouses would find it harder to find jobs in OCONUS sites due to language and cultural barriers.

Table 9. Pay-off Matrix for the Family Life Preference Factor in the Sailor Utility Function

Billet Location	**Representative City**	**Recent Job Growth Rates[3] (National Average = 1.88%)**	**Family Life Factor**	**Score**
Gulf Coast (CGC)	Pensacola	3.94%	Excellent	5
East Coast (CEC)	Norfolk	2.14%	High	4
Southwest (CSW)	San Diego	2.10%	Average	3
Northwest (CNW)	Bremerton	0.91%	Moderate	2
OCONUS (OPL)	--	--	Low	1

Location Factor – The location factor is determined by the affordability of a house in the billet location, whether the location has an affordable cost of living and how easy it is to

[3] The percentage increase or decrease in available jobs over the most recent 12-month period for the cities of Pensacola, Norfolk, San Diego, Bremerton, as obtained from http://www.homeadvisor.msn.com (October, 2002).

transit to civilian life in that location. The affordability of owning a house and how affordable the cost of living is can be captured by the overall cost of living index of the location. The ease of transit to civilian life can be captured by the recent job growth rates of the billet location. The location factor will be proxied by the overall cost of living index of the representative city in the billet location since the recent job growth rates are already captured by the family life factor. A score of 5 is given to a job that has excellent location qualities and a score of 1 is given to a job that has low location qualities. As the OCONUS locations are highly diverse and aggregate data would not be representative, it is assumed that half the OCONUS locations have a lower of cost of living than the US and half have a higher cost of living than the US which gives it a score of 3 for the location factor.

Table 10. Pay-off Matrix for the Location Preference Factor in the Sailor Utility Function

Billet Location	**Representative City**	**Overall Cost of Living Index[4] (National Average = 100)**	**Location Factor**	**Score**
Gulf Coast (CGC)	Pensacola	94.9	Excellent	5
East Coast (CEC)	Norfolk	96.6	High	4
OCONUS (OPL)	--	--	Average	3
Northwest (CNW)	Bremerton	100	Moderate	2
Southwest (CSW)	San Diego	136.4	Low	1

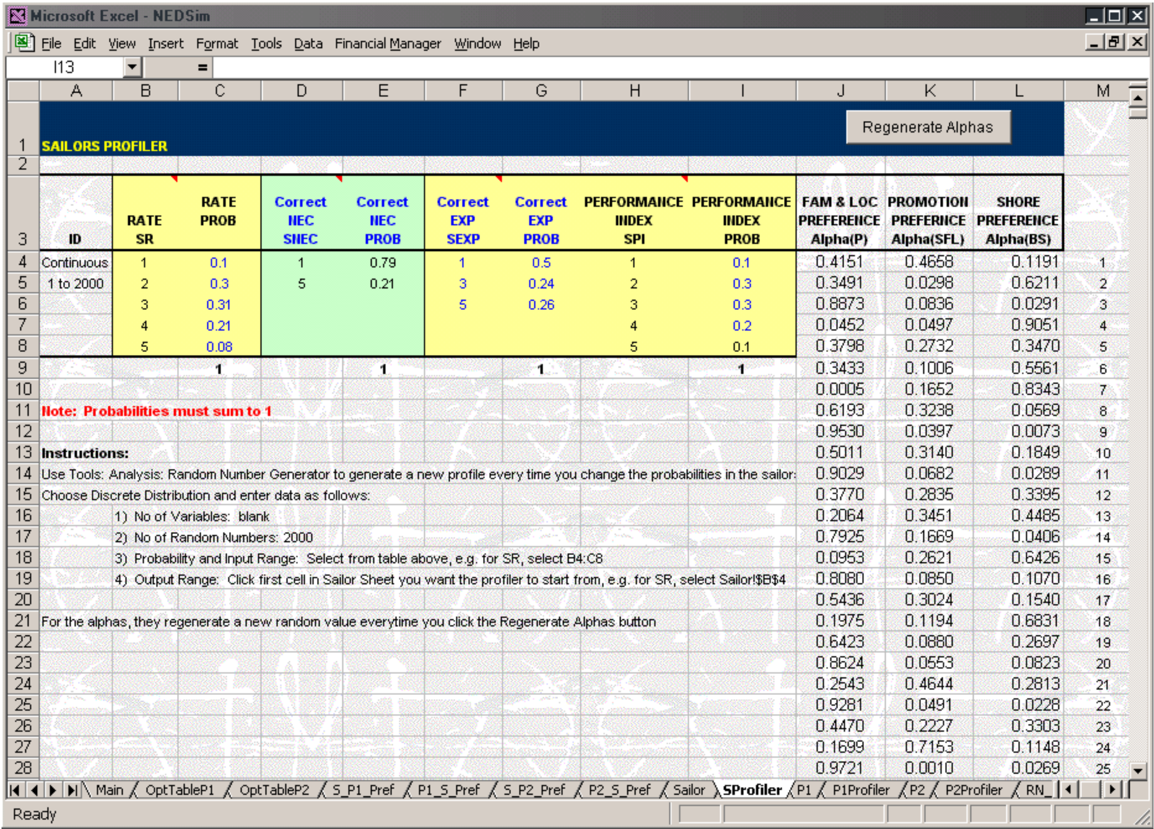
Job Factor – The job factor can be split into two sub-factors, consisting of the billets' visibility and whether the billet is on shore or at sea. The billets' visibility ranges from low to excellent, and corresponding scores from 1 to 5 are assigned. The sailor will also derive greater satisfaction from a shore billet and is assigned a score of 5 whilst the sea billet will be assigned a value of 1, corresponding to a lower satisfaction level.

[4] The total of all the cost of living categories weighted subjectively as follows: housing (30%), food and groceries (15%), transportation (10%), utilities (6%), health care (7%), and miscellaneous expenses such as clothing, services, and entertainment (32%). State and local taxes are not included. Information is obtained from http://www.homeadvisor.msn.com (Oct, 2002).

Table 11. Pay-off Matrix for the Job Preference Factor in the Sailor Utility Function

Billet Visibility	**Score**	**Billet Sea or Shore**	**Score**
Excellent (Billet rate ≥ 2 rates above Sailor rate)	5	Shore	5
High (Billet rate = 1 rate above Sailor rate)	4	Sea	1
Average (Billet rate = Sailor rate)	3		
Low (Billet rate = 1 rate below Sailor rate)	2		
Extremely Low (Billet rate ≤ 2 rates below Sailor rate)	1		

In reality, sailors are not allowed to apply for billets more than one paygrade from the sailors' rate. For the purpose of the simulation, however, NEDSim permits sailors to apply for all jobs, regardless of the billets' rating. Sailors who do apply for such out-of-range jobs will be much less attractive to the billets they applied for. As such, these billets will derive a low utility level from these sailors. This will push the sailors down the billets' preference list and effectively "prohibits" such applications. The simulator then generates the Sailor's Preference List by calculating the Sailor Utility (U_s) for each eligible billet and ranks the billets in decreasing order of utility. Figure 18 shows the "SProfiler" worksheet that incorporates the Sailor's characteristics and the weightage that each Sailor assigns to the billets' characteristics.



Microsoft Excel - NEDSim

	ID	RATE SR	RATE PROB	Correct NEC SNEC	Correct NEC PROB	Correct EXP SEXP	Correct EXP PROB	PERFORMANCE INDEX SPI	PERFORMANCE INDEX PROB	FAM & LOC PREFERENCE Alpha(P)	PROMOTION PREFERENCE Alpha(SFL)	SHORE PREFERENCE Alpha(BS)	
1	SAILORS PROFILER										Regenerate Alphas		
4	Continuous	1	0.1	1	0.79	1	0.5	1	0.1	0.4151	0.4658	0.1191	1
5	1 to 2000	2	0.3	5	0.21	3	0.24	2	0.3	0.3491	0.0298	0.6211	2
6		3	0.31			5	0.26	3	0.3	0.8873	0.0836	0.0291	3
7		4	0.21					4	0.2	0.0452	0.0497	0.9051	4
8		5	0.08					5	0.1	0.3798	0.2732	0.3470	5
9			1		1		1		1	0.3433	0.1006	0.5561	6
10										0.0005	0.1652	0.8343	7
11	Note: Probabilities must sum to 1									0.6193	0.3238	0.0569	8
12										0.9530	0.0397	0.0073	9
13	Instructions:									0.5011	0.3140	0.1849	10
14	Use Tools: Analysis: Random Number Generator to generate a new profile every time you change the probabilities in the sailor.									0.9029	0.0682	0.0289	11
15	Choose Discrete Distribution and enter data as follows:									0.3770	0.2835	0.3395	12
16		1) No of Variables: blank								0.2064	0.3451	0.4485	13
17		2) No of Random Numbers: 2000								0.7925	0.1669	0.0406	14
18		3) Probability and Input Range: Select from table above, e.g. for SR, select B4:C8								0.0953	0.2621	0.6426	15
19		4) Output Range: Click first cell in Sailor Sheet you want the profiler to start from, e.g. for SR, select Sailor!B4								0.8080	0.0850	0.1070	16
20										0.5436	0.3024	0.1540	17
21	For the alphas, they regenerate a new random value everytime you click the Regenerate Alphas button									0.1975	0.1194	0.6831	18
22										0.6423	0.0880	0.2697	19
23										0.8624	0.0553	0.0823	20
24										0.2543	0.4644	0.2813	21
25										0.9281	0.0491	0.0228	22
26										0.4470	0.2227	0.3303	23
27										0.1699	0.7153	0.1148	24
28										0.9721	0.0010	0.0269	25

Main / OptTableP1 / OptTableP2 / S_P1_Pref / P1_S_Pref / S_P2_Pref / P2_S_Pref / Sailor / SProfiler / P1 / P1Profiler / P2 / P2Profiler / RN_

Ready

Figure 18. "SProfiler" Worksheet to capture Sailor's characteristics

b. Command Preference Lists

The preference factors that are deemed important to the command include sailor NEC, sailor paygrade, sailor specific experience and the sailor's performance. The command's total utility for the job is then modeled by the following utility function:

$$\text{Command Utility} = \beta_{NEC}(\text{NEC}) + \beta_{pay}(\text{Paygrade}) + \beta_{exp}(\text{Experience}) + \beta_{perf}(\text{Performance})$$

where : $\beta_{FL} + \beta_L + \beta_{J1} + \beta_{J2} = 1$

(each β is generated randomly)

NEC – Sailors who have the same NECs as the billet are given a score of 5, and sailors with NECs that are different from the billet are given a score of 1.

Paygrade – The sailor's paygrade is restricted to those up to one above and one below the billet paygrade. Sailors having the correct paygrade are given a score of 5, sailors who have a paygrade which is one above the billet are given a score of 3, and sailors with a paygrade which is one below the billet are the least preferred and given a score of 1.

Experience – The level of experience is separated into three levels. Sailors who are from the same platform or type of unit as the billet to which they are going are given a score of 5. Sailors who are from sea billets and going to another sea billet, or from shore billets and going to another shore billet, are given a score of 3, and sailors who are in a billet that is different from the billet to which they are going are given a score of 1.

Performance – The performance of the sailor is separated into five categories of early promote, must promote, promotable, progressing and not promote listed in decreasing order of preference and given scores of 5, 4, 3, 2 and 1, respectively. The distribution of performance scores is assumed to be normal.

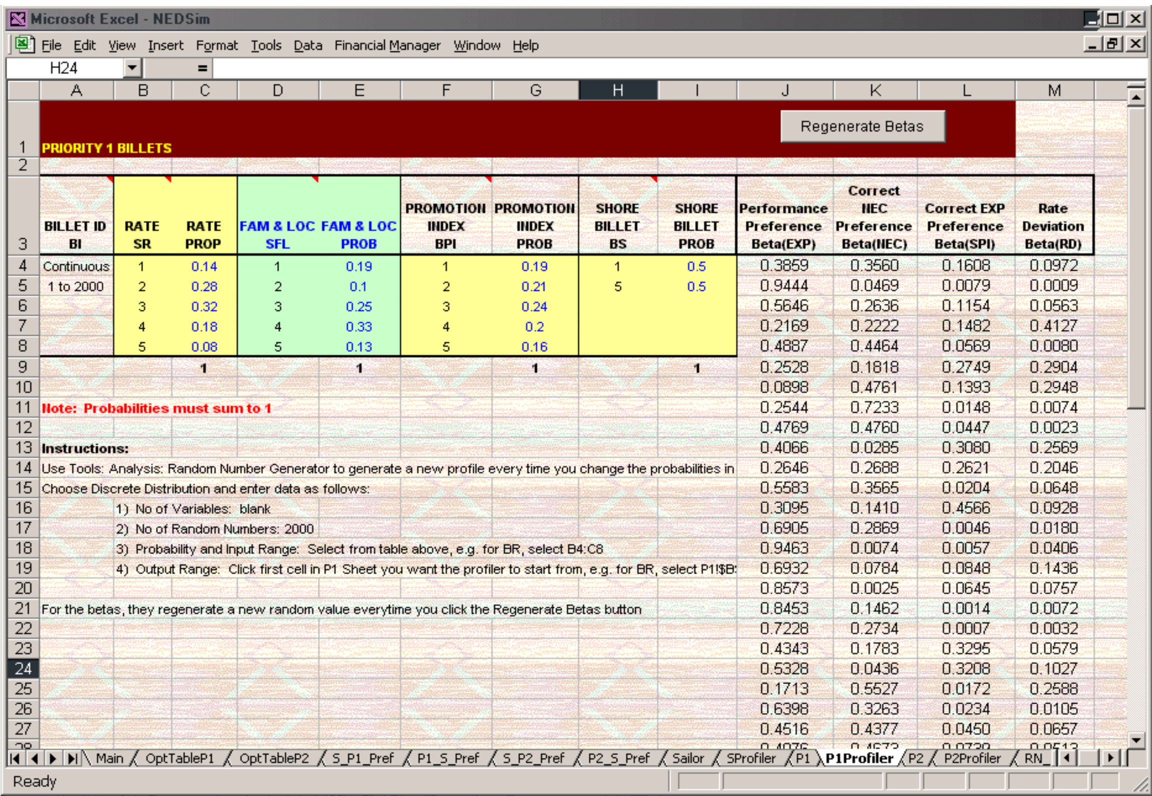
A summary of the pay-off matrix for the billet characteristics is given in the following table.

Table 12. Pay-off Matrix for Billet Characteristics

NEC	Paygrade	Experience	Performance	Score
Correct (Sailor NEC = Billet NEC)	Correct	Highly relevant (Sailor Unit = Billet Unit)	Early Promote	5
--	--	--	Must Promote	4
--	One above	Somewhat relevant (Sailor Sea/Shore = Billet Sea/Shore)	Promotable	3
--	--	--	Progressing	2
Incorrect (Sailor NEC ≠ Billet NEC)	One below	Not relevant (Sailor Sea/Shore ≠ Billet Sea/Shore)	Not Promote	1

Using the values in the pay-off matrices, NEDSim then generates the Command's Preference List by calculating the Command Utility (U_c) for each eligible sailor and ranks the sailors in decreasing order of utility.

Figure 19 shows the "P1Profiler" worksheet that incorporates the P1 Billet's characteristics and the weights that each billet assigns to the respective Sailor characteristics.



Microsoft Excel - NEDSim

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	PRIORITY 1 BILLETS									Regenerate Betas			
2													
3	BILLET ID BI	RATE SR	RATE PROP	FAM & LOC SFL	FAM & LOC PROB	PROMOTION INDEX BPI	PROMOTION INDEX PROB	SHORE BILLET BS	SHORE BILLET PROB	Performance Preference Beta(EXP)	Correct NEC Preference Beta(NEC)	Correct EXP Preference Beta(SPI)	Rate Deviation Beta(RD)
4	Continuous	1	0.14	1	0.19	1	0.19	1	0.5	0.3859	0.3560	0.1608	0.0972
5	1 to 2000	2	0.28	2	0.1	2	0.21	5	0.5	0.9444	0.0469	0.0079	0.0009
6		3	0.32	3	0.25	3	0.24			0.5646	0.2636	0.1154	0.0563
7		4	0.18	4	0.33	4	0.2			0.2169	0.2222	0.1482	0.4127
8		5	0.08	5	0.13	5	0.16			0.4887	0.4464	0.0569	0.0080
9			1		1		1		1	0.2528	0.1818	0.2749	0.2904
10										0.0898	0.4761	0.1393	0.2948
11	Note: Probabilities must sum to 1									0.2544	0.7233	0.0148	0.0074
12										0.4769	0.4760	0.0447	0.0023
13	Instructions:									0.4066	0.0285	0.3080	0.2569
14	Use Tools: Analysis: Random Number Generator to generate a new profile every time you change the probabilities in									0.2646	0.2688	0.2621	0.2046
15	Choose Discrete Distribution and enter data as follows:									0.5583	0.3565	0.0204	0.0648
16		1) No of Variables: blank								0.3095	0.1410	0.4566	0.0928
17		2) No of Random Numbers: 2000								0.6905	0.2869	0.0046	0.0180
18		3) Probability and Input Range: Select from table above, e.g. for BR, select B4:C8								0.9463	0.0074	0.0057	0.0406
19		4) Output Range: Click first cell in P1 Sheet you want the profiler to start from, e.g. for BR, select P1!$B:								0.6932	0.0784	0.0848	0.1436
20										0.8573	0.0025	0.0645	0.0757
21	For the betas, they regenerate a new random value everytime you click the Regenerate Betas button									0.8453	0.1462	0.0014	0.0072
22										0.7228	0.2734	0.0007	0.0032
23										0.4343	0.1783	0.3295	0.0579
24										0.5328	0.0436	0.3208	0.1027
25										0.1713	0.5527	0.0172	0.2588
26										0.6398	0.3263	0.0234	0.0105
27										0.4516	0.4377	0.0450	0.0657

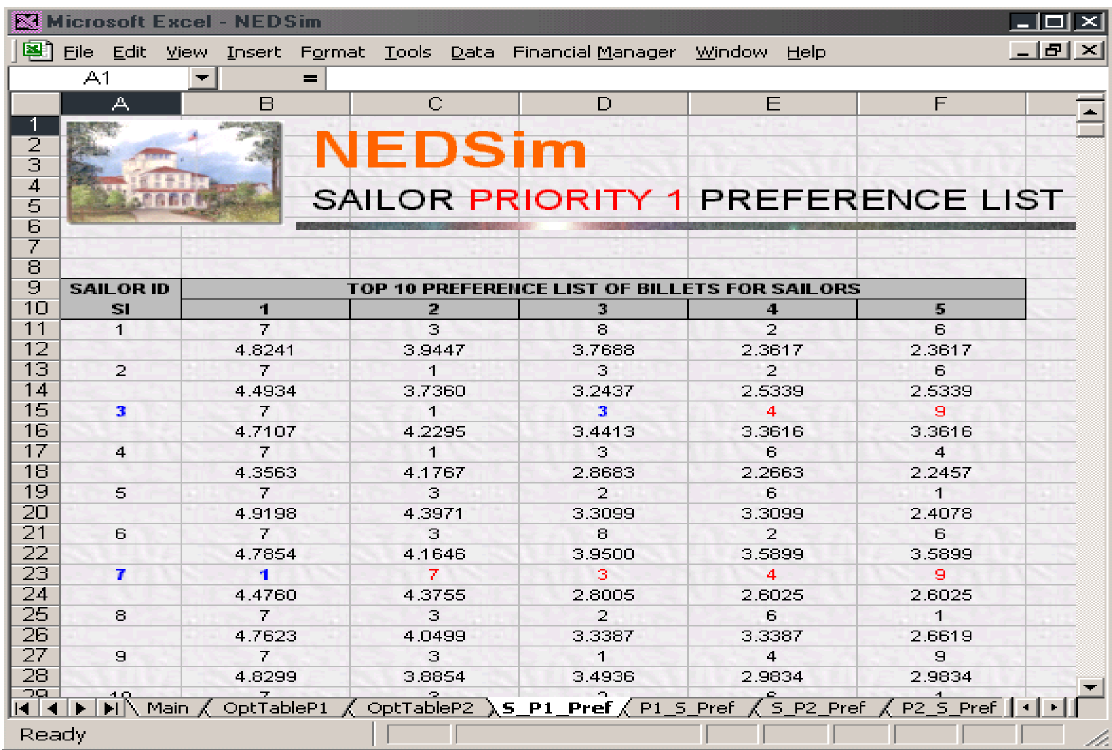
Main / OptTableP1 / OptTableP2 / S_P1_Pref / P1_S_Pref / S_P2_Pref / P2_S_Pref / Sailor / SProfiler / P1 / P1Profiler / P2 / P2Profiler / RN_

Ready

Figure 19. "P1Profiler" Worksheet to capture P1 billet's characteristics

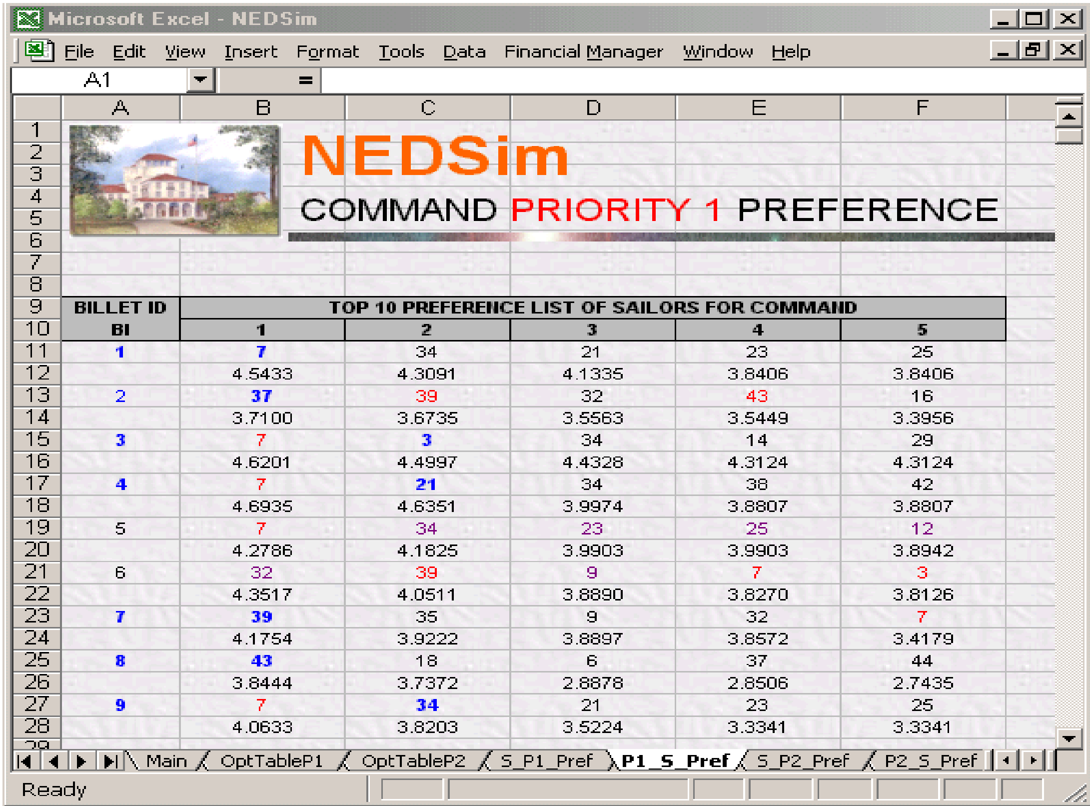
In the preference list generation, the agent will prioritize the sailors' and commands' preference lists according to the returned sailor utility, U_S, and command utility, U_C. The preference lists are kept in worksheets *S_P1_Pref* (Sailors to P1 billets), *P1_S_Pref* (P1 billets to sailors), *S_P2_Pref* (Remaining unmatched sailors to P2/P3 billets), and *P2_S_Pref* (P2/P3 billets to remaining sailors). Figure 20 shows that billet 7 offers the highest utility to sailor 1 (utility of 4.8241), followed by billet 3 with a value of

3.9447. Similarly, priority 1 commands have their preference list of sailors. For example, in Figure 21, sailor 7 offers the highest utility to billet 1 (utility of 4.5433), followed by sailor 34 with a value of 4.3091. The agent then uses the chosen matching logic (sailor or command biased) to find the best stable matches, while discarding the unstable matches. The matched sailors and billets are shown in blue. Thus, in the sample shown below, sailor 3 is matched to billet 3. Sailor 2 is unmatched because there are no stable matches found. The unmatched sailors will go for a second round generation of preference lists and matching logic with priority 2/3 requisition billets.



SAILOR ID	TOP 10 PREFERENCE LIST OF BILLETS FOR SAILORS				
SI	1	2	3	4	5
1	7	3	8	2	6
	4.8241	3.9447	3.7688	2.3617	2.3617
2	7	1	3	2	6
	4.4934	3.7360	3.2437	2.5339	2.5339
3	7	1	3	4	9
	4.7107	4.2295	3.4413	3.3616	3.3616
4	7	1	3	6	4
	4.3563	4.1767	2.8683	2.2663	2.2457
5	7	3	2	6	1
	4.9198	4.3971	3.3099	3.3099	2.4078
6	7	3	8	2	6
	4.7854	4.1646	3.9500	3.5899	3.5899
7	1	7	3	4	9
	4.4760	4.3755	2.8005	2.6025	2.6025
8	7	3	2	6	1
	4.7623	4.0499	3.3387	3.3387	2.6619
9	7	3	1	4	9
	4.8299	3.8854	3.4936	2.9834	2.9834

Figure 20. Sailors' Preferences for Priority 1 Billets generated by NEDSim



BILLET ID	TOP 10 PREFERENCE LIST OF SAILORS FOR COMMAND				
BI	1	2	3	4	5
1	7	34	21	23	25
	4.5433	4.3091	4.1335	3.8406	3.8406
2	37	39	32	43	16
	3.7100	3.6735	3.5563	3.5449	3.3956
3	7	3	34	14	29
	4.6201	4.4997	4.4328	4.3124	4.3124
4	7	21	34	38	42
	4.6935	4.6351	3.9974	3.8807	3.8807
5	7	34	23	25	12
	4.2786	4.1825	3.9903	3.9903	3.8942
6	32	39	9	7	3
	4.3517	4.0511	3.8890	3.8270	3.8126
7	39	35	9	32	7
	4.1754	3.9222	3.8897	3.8572	3.4179
8	43	18	6	37	44
	3.8444	3.7372	2.8878	2.8506	2.7435
9	7	34	21	23	25
	4.0633	3.8203	3.5224	3.3341	3.3341

Figure 21. Priority 1 Commands' Preferences for Sailors Generated by NEDSim

3. Matching Algorithm

NEDSim highlights two different matching algorithms: (1) the Deferred Acceptance (DA) algorithm and the (2) Linear Programming (LP) algorithm.

a. DA Algorithm

In the DA matching algorithm, the sailor and billets are like the interns and consultants as described in Section A.1 of Chapter 3. The result of assignment varies slightly depending on whether a Sailor or Command-biased logic is employed. For example, the user can activate the command-biased logic by pressing the "*Command-biased Matching Logic*" button. This triggers the simulator to start the process by first going through each billet's preference lists, and then executing the iteration steps described in Chapter 3 until none of the shortlisted sailors are rejected. This will result in a Command-biased match. The commands' utilities (but not necessarily both the sailors' and commands' utility) are

maximized while still ensuring a stable match. The match will be an optimal match for the commands as a group. Similar action will result if the user opts for a Sailor-biased logic by activating the "*Sailor-biased Matching Logic*" button.

b. LP Algorithm

The LP problem can generally be tackled by invoking the Solver add-in within Excel. However, the standard Solver add-in only caters to a maximum of 200 decision variables; the sailors/billet mix using a one-month requisition cycle would be 90/120 or potentially create 10,800 decision variables. To accommodate this larger problem size, an enhanced version of the Solver engine: Premium Solver Platform version 3.5 and the Large Scale LP Solver engine version 4.0 are employed to ensure a smooth simulation run. Two special worksheets: "OptTableP1" and "OptTableP2" are created to process the LP calculation. Figure 22 shows a snapshot of the worksheet "OptTableP1," where the top half of the worksheet is the shaded table representing the dichotomic (0,1) matrix (Sailors and Command numbers are represented row and column-wise, respectively). The constraints are indicated by one row and column each to reflect the condition that there can only be one unique match per Sailor or Command.

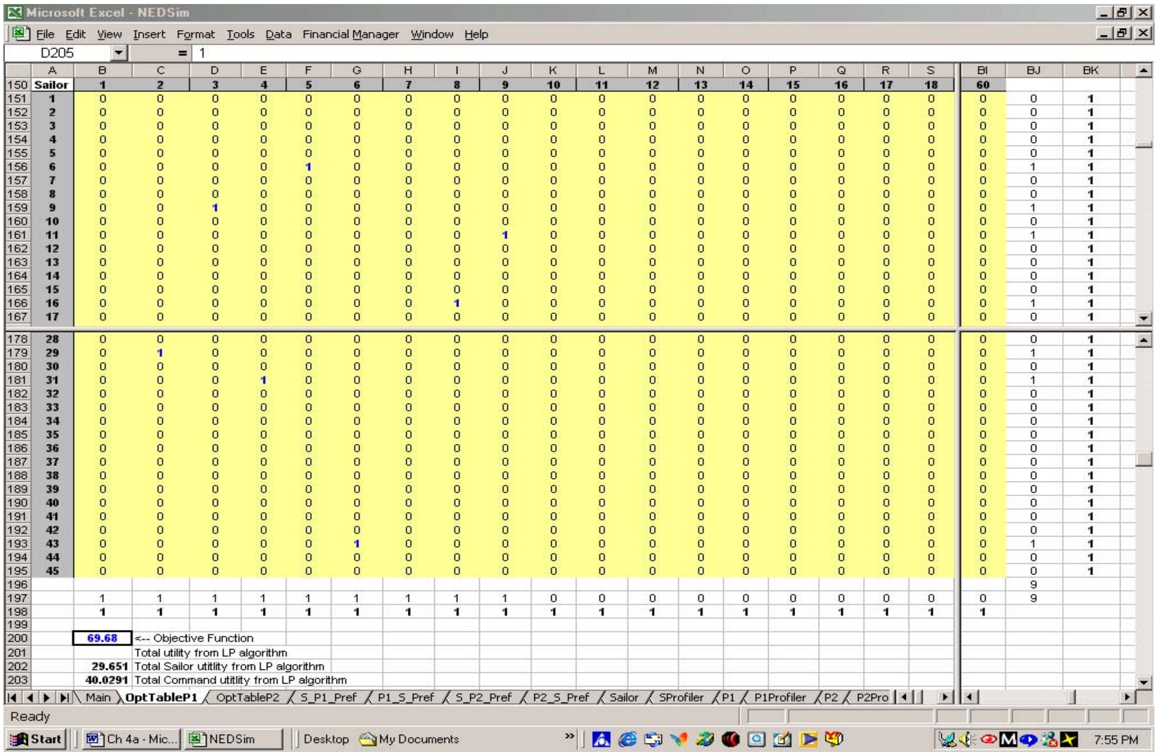


Figure 22. “OptTableP1” worksheet - Optimization table

c. Check for Blocking Pairs or Unstable Matches

The check for blocking pairs is done using two steps:

(1) for each sailor that is matched, determine the rank position of their matching billet. If the matched billet was ranked first in the sailors’ list, we can conclude that this is a stable match and there is no need to proceed further. The sailors would not want to break the match since they have been assigned to their most preferred billet. This is true even if the matched billet ranked them lower than other sailors in the billet’s lists. A blocking pair can only occur if both parties are willing and “collude” to break the match;

(2) if the matched billet is not the sailors’ most preferred choice, then all of the higher-ranked billets in their list are examined to see if any of these billets have ranked that sailor higher than the sailor that was assigned by the optimization. If there is such a case, a blocking pair is registered. Figure 12 shows another area of the worksheet “OptTableP1” that incorporates these procedures.

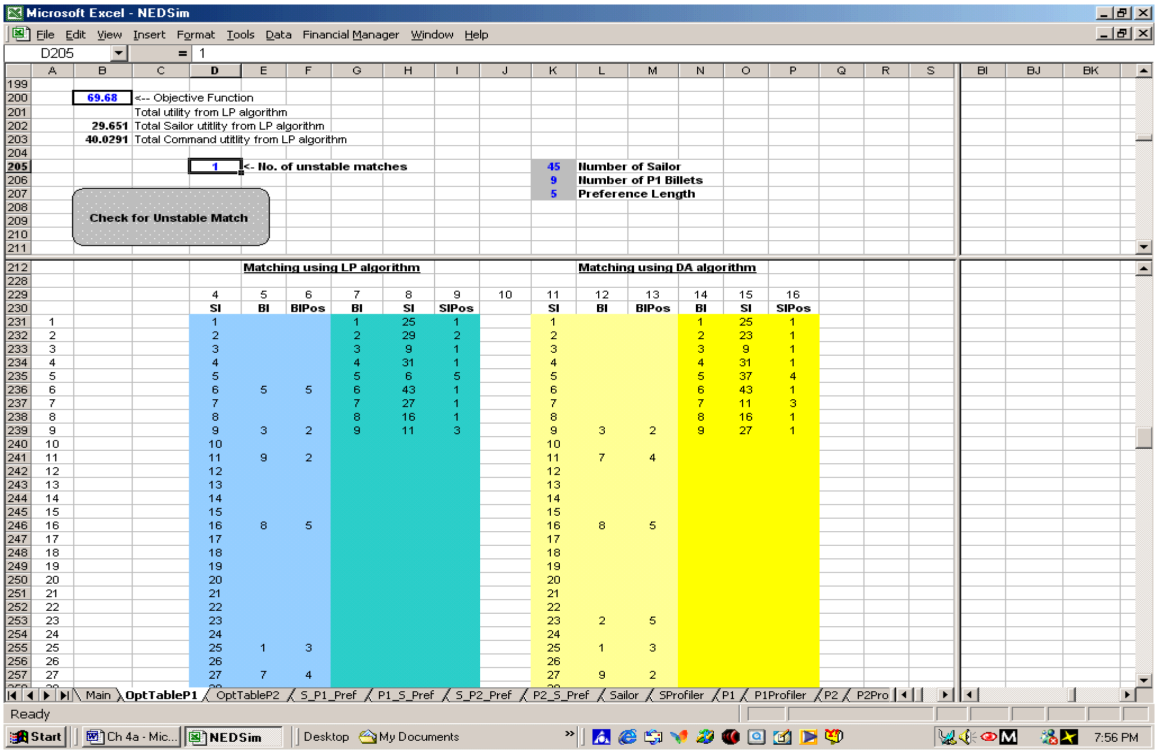
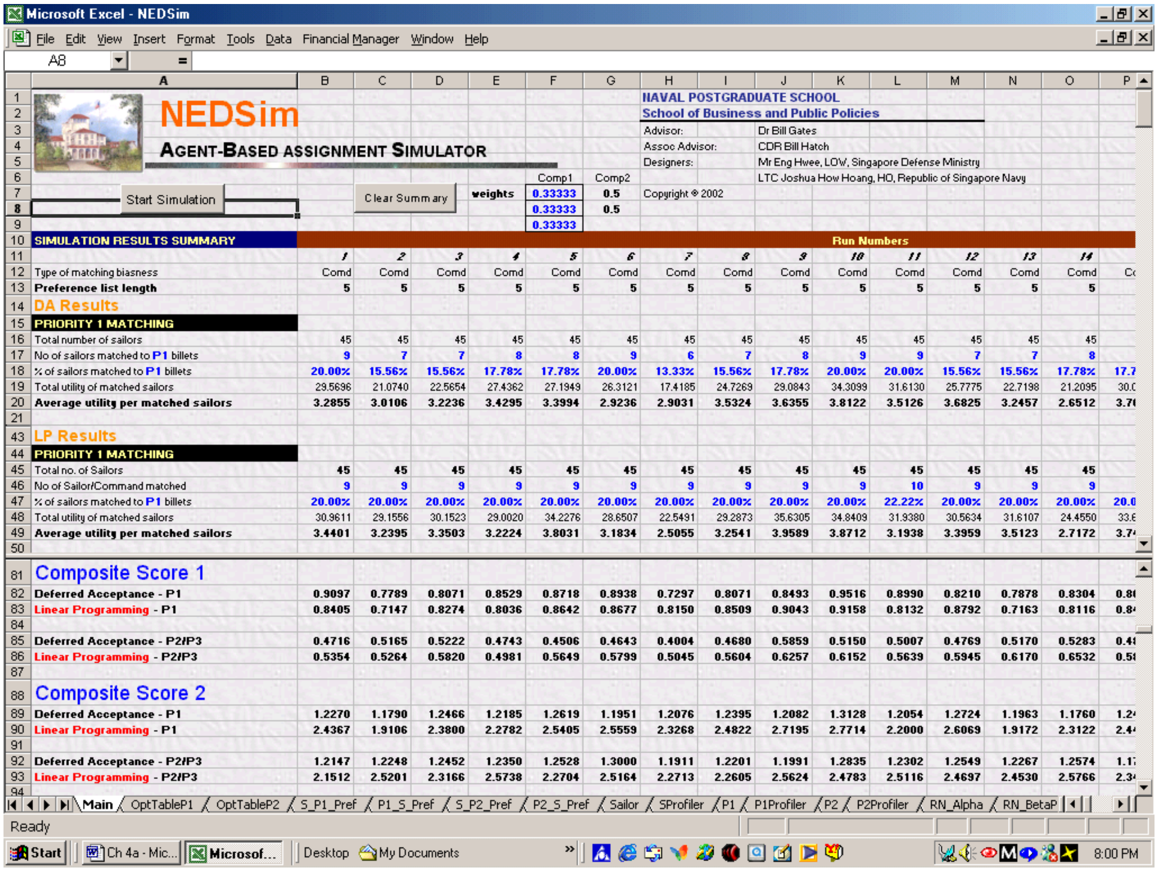


Figure 23. “OptTableP1” worksheet - matching table and “check unstable matching” button

4. NEDSim Output

After a successful simulation run, ABEMS will output the results of the simulation in the main worksheet, as shown in Figure 24 below. The output primarily comprises the *qualitative* and *quantitative performance indicators* of the matching for both P1 and P2/P3 billets, as described in Section B of this Chapter.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Type of matching biasness	Comd	Comd	Comd	Comd	Comd	Comd	Comd	Comd	Comd	Comd	Comd	Comd	Comd	Comd
Preference list length	5	5	5	5	5	5	5	5	5	5	5	5	5	5
DA Results														
PRIORITY 1 MATCHING														
Total number of sailors	45	45	45	45	45	45	45	45	45	45	45	45	45	45
No of sailors matched to P1 billets	9	7	7	8	8	9	6	7	8	9	9	7	7	8
% of sailors matched to P1 billets	20.00%	15.56%	15.56%	17.78%	17.78%	20.00%	13.33%	15.56%	17.78%	20.00%	20.00%	15.56%	15.56%	17.78%
Total utility of matched sailors	29.5696	21.0740	22.5654	27.4362	27.1949	26.3121	17.4185	24.7269	29.0843	34.3099	31.6130	25.7775	22.7198	21.2095
Average utility per matched sailors	3.2855	3.0106	3.2236	3.4295	3.3994	2.9236	2.9031	3.5324	3.6355	3.8122	3.5126	3.6825	3.2457	2.6512
LP Results														
PRIORITY 1 MATCHING														
Total no. of Sailors	45	45	45	45	45	45	45	45	45	45	45	45	45	45
No of Sailor/Command matched	9	9	9	9	9	9	9	9	9	9	10	9	9	9
% of sailors matched to P1 billets	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	20.00%	22.22%	20.00%	20.00%	20.00%
Total utility of matched sailors	30.9611	29.1556	30.1523	29.0020	34.2276	28.6507	22.5491	29.2873	35.6305	34.8409	31.9380	30.5634	31.6107	24.4550
Average utility per matched sailors	3.4401	3.2395	3.3503	3.2224	3.8031	3.1834	2.5055	3.2541	3.9589	3.8712	3.1938	3.3959	3.5123	2.7172
Composite Score 1														
Deferred Acceptance - P1	0.9097	0.7789	0.8071	0.8529	0.8718	0.8938	0.7297	0.8071	0.8493	0.9516	0.8990	0.8210	0.7878	0.8304
Linear Programming - P1	0.8405	0.7147	0.8274	0.8036	0.8642	0.8677	0.8150	0.8509	0.9043	0.9158	0.8132	0.8792	0.7163	0.8116
Deferred Acceptance - P2/P3	0.4716	0.5165	0.5222	0.4743	0.4506	0.4643	0.4004	0.4680	0.5859	0.5150	0.5007	0.4769	0.5170	0.5283
Linear Programming - P2/P3	0.5354	0.5264	0.5820	0.4981	0.5649	0.5799	0.5045	0.5604	0.6257	0.6152	0.5639	0.5945	0.6170	0.6532
Composite Score 2														
Deferred Acceptance - P1	1.2270	1.1790	1.2466	1.2185	1.2619	1.1951	1.2076	1.2395	1.2082	1.3128	1.2054	1.2724	1.1963	1.1760
Linear Programming - P1	2.4367	1.9106	2.3800	2.2782	2.5405	2.5559	2.3268	2.4822	2.7195	2.7714	2.2000	2.6069	1.9172	2.3122
Deferred Acceptance - P2/P3	1.2147	1.2248	1.2452	1.2350	1.2528	1.3000	1.1911	1.2201	1.1991	1.2835	1.2302	1.2549	1.2267	1.2574
Linear Programming - P2/P3	2.1512	2.5201	2.3166	2.5738	2.2704	2.5164	2.2713	2.2605	2.5624	2.4783	2.5116	2.4697	2.4530	2.5766

Figure 24. Output page of NEDSim

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V. NEDSIM RESULTS & FINDINGS

A. GENERAL DESCRIPTION

NEDSim models the current USN detailing process for the AS community using a 2-week requisition cycle. It is used to generate empirical data to compare the DA and LP outcomes based on their advantages and disadvantages. In particular, the study examines the trade-offs between stable matching but potentially lower utility under the DA algorithm and the potentially higher utility but unstable matching under the LP method. Both algorithms are assessed by the quantity as well as the quality of the matches produced. The measure used to determine the quantity of matches produced is the percent matched for the sailors, priority 1 (P1) billets and priority 2/3 (P2/3) billets. The measures used to determine the quality of matches produced are the sailors' or commands' utility and percentage of unstable matches.

Under the 2-week requisition cycle, typically 60 billets are available for consideration while 45 sailors are available to be assigned. In addition, the proportion of priority 1 billets out of the total number available for assignment is estimated to be 15%. The assumptions were taken from previous simulation runs conducted by Ng and Soh (2001), based on data compiled by Short (2000).

In the simulation, the preference length is kept constant at the current norm of five, because we believe that it is impractical for sailors to consider more than five potential choices. While the simulation model is able to cater to a different requisition cycle, it is our intent to keep to the current arrangement. As such, the number of sailors, P1 billets and P2/3 billets number are fixed at 45, 9 and 51, respectively, to reflect the current 2-week requisition cycle and P1:P2/3 ratio. The sailor and command profiles were created through a random process. The utility scores for the preference factors of rating, NEC, location, relevant training, and promotion prospects are generated based on a derived distribution. The derived distribution is developed from a previous survey on what the sailors and commands consider important preference factors and from the actual distribution of sailor

and billet characteristics obtained from master files for the AS community. The utility score for the correct rate is derived indirectly by comparing the paygrade and rate profile of sailors and billets. In addition, the weights assigned to each of these preference factors for the sailors and billets are generated randomly to simulate the relative worth of each factor to the individual sailor or command. For the thesis, NEDSim generated data for one hundred simulation runs, which is equivalent to four years equivalent of requisition cycles.

B. GENERAL COMPARISON OF STATISTICS BETWEEN DA & LP

The results for the quantity and quality of matches from the simulation are presented in this section. The quantity of matches is measured by the number of sailors or billets matched as a proportion of all available sailors and billets, and indicates the efficiency of the algorithm adopted. On the other hand, the quality of matches is measured by the utility derived from each successful match and whether such matching is vulnerable to alternative private pairing arrangements. The quality of matches indicates the effectiveness of the algorithm adopted.

1. Comparison of Percentage Matched

Figure 25 shows the sailor and command percentage matched for P1 billets. The percentage of sailors matched is lower than the percentage of commands (billets) matched, as the number of P1 billets (9) is much smaller than the number of sailors (45). It is possible for up to 100% of commands to be matched, but only up to a maximum of 20% (9/45) of sailors to be matched. The results also show that LP has a higher percentage of matches than DA. Table 13 shows that the percentage of matches for sailors is 2.1% higher under LP than DA and the percentage of matches for commands is 10.6% higher under LP than DA. The results are significant at the 0.05 level using a two-sample t-test on the means, assuming unequal population variances for the DA and the LP data.

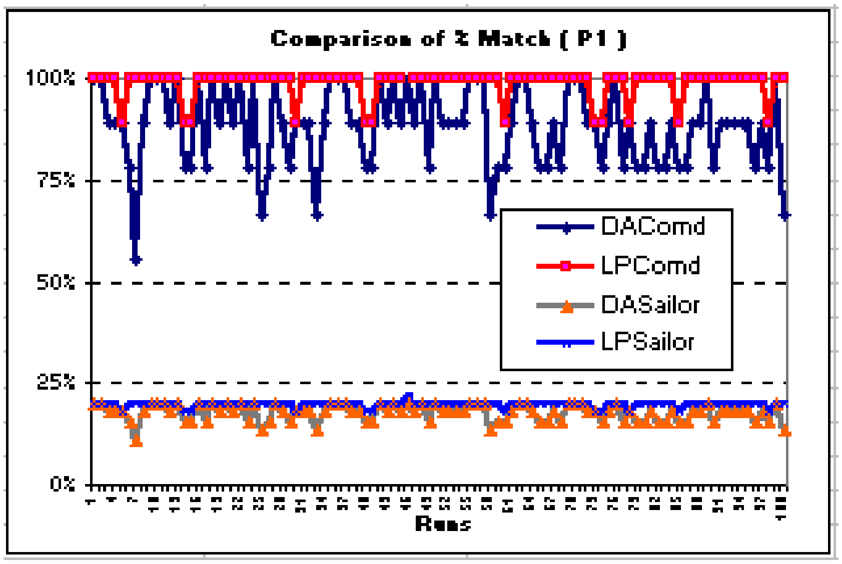

Figure 25. Sailor and Command Percentage Matched (P1 billets)

Table 13. Two-Sample t-test on Means Assuming Unequal Variances for P1 Billets (Percentage Matched)

	Sailor		***Command***	
	DA	***LP***	***DA***	***LP***
Mean	0.177	0.198	0.882	0.988
Variance	0.000407	5.869E-05	0.0102	0.00147
Observations	100	100	100	100
Hypothesized Mean Difference	0		0	
df	127		127	
t Stat	-9.776		-9.779	
P(T<=t) one-tail	1.794E-17		1.769E-17	
t Critical one-tail	1.657		1.657	
P(T<=t) two-tail	3.588E-17		3.538E-17	
t Critical two-tail	1.979		1.979	

For P2/3 billets (see Figures 26 and 27), the LP outcomes are also favorable compared to the DA method, and the difference in outcomes is even more pronounced than with P1 billets. The percentage of billets being matched is lower than that for the sailors, because the number of P2/3 billets (51) is now greater than the number of sailors (36). It is possible for up to 100% of sailors to be matched, whilst it is only possible for up to 71% of billets to be matched (36/51), assuming nine sailors are matched to P1 billets. Table 14 shows that the percentage of matches for sailors is 16.2% higher under LP than DA and the

percentage of matches for commands is 11.7% higher under LP than DA. The results are significant at the 0.05 level using a two-sample t-test on the means, assuming unequal population variances for the DA and the LP data.

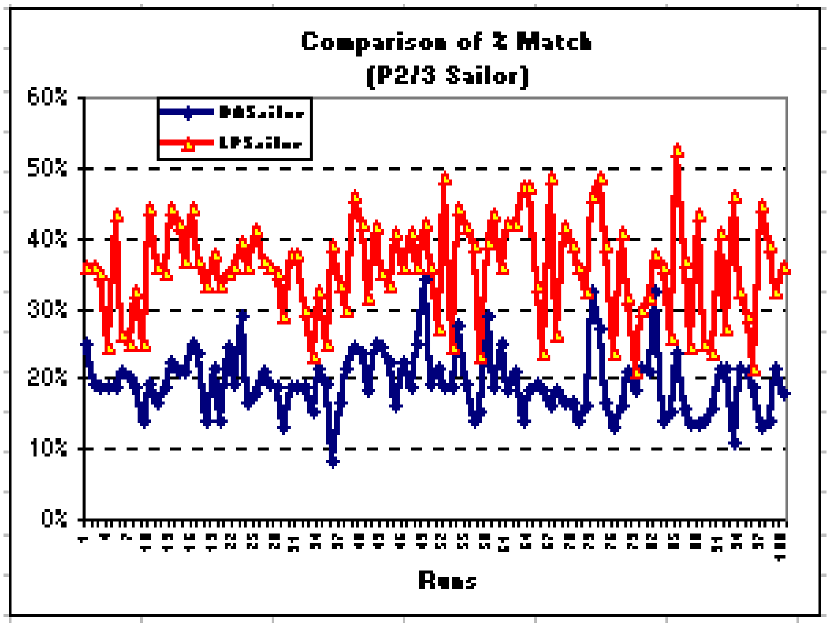

Figure 26. Sailor Percentage Matched (P2/3 billets)

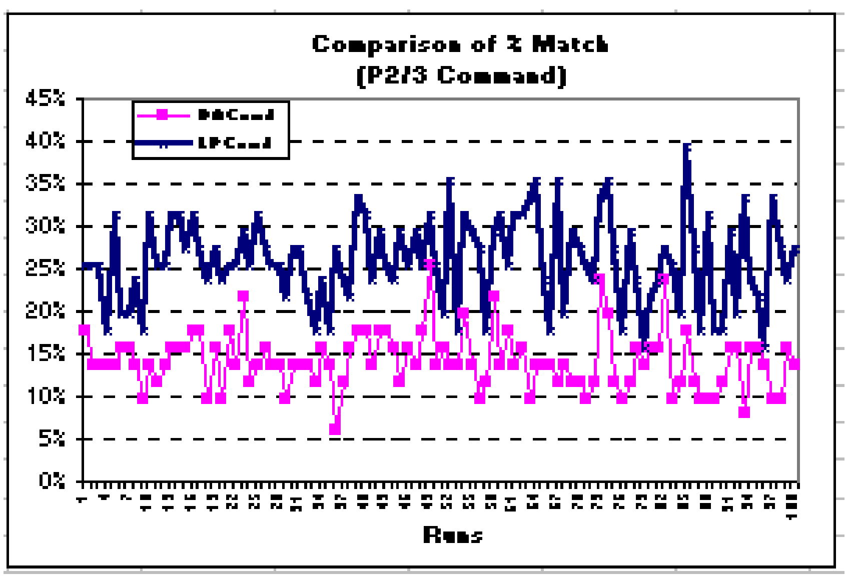

Figure 27. Command Percentage Matched (P2/3 billets)

Table 14. Two-Sample t-test on Means for P2/3 Billets Assuming Unequal Variances (Percentage Matched)

	Sailor		***Command***	
	DA	***LP***	***DA***	***LP***
Mean	0.195	0.357	0.142	0.259
Variance	0.00209	0.00529	0.00113	0.00273
Observations	100	100	100	100
Hypothesized Mean Difference	0		0	
df	167		169	
t Stat	-18.898		-18.917	
P(T<=t) one-tail	1.242E-43		6.978E-44	
t Critical one-tail	1.654		1.654	
P(T<=t) two-tail	2.485E-43		1.396E-43	
t Critical two-tail	1.974		1.974	

2. Comparison of Utility

The first measure of quality is the utility that both sailors and commands derive from any matching pair. We first discuss the difference in *total utility*. Figure 28 shows the combined utility of the matched sailors and P1 billets for both the DA and LP method. For the matching process involving P1 billets, the LP method results in higher *total utility*. Figure 29 shows the total utility of the sailors and P2/3 billets for both the DA and LP method. Again the LP method results in higher *total utility*. Based on the mean value of total utility, the LP's outcome is 9.7% higher than the DA's result for the process involving P1 billets, compared to the substantial 46.9% difference for the process involving P2/3 billets (Table 15) with the results significant at the 0.05 level.

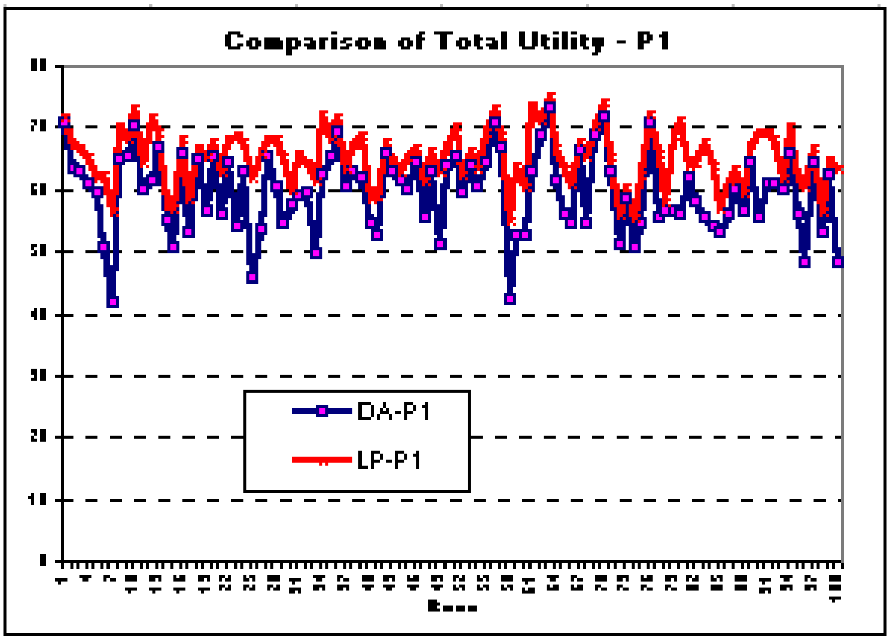

Figure 28. Total Utility of Matches (P1 billets)

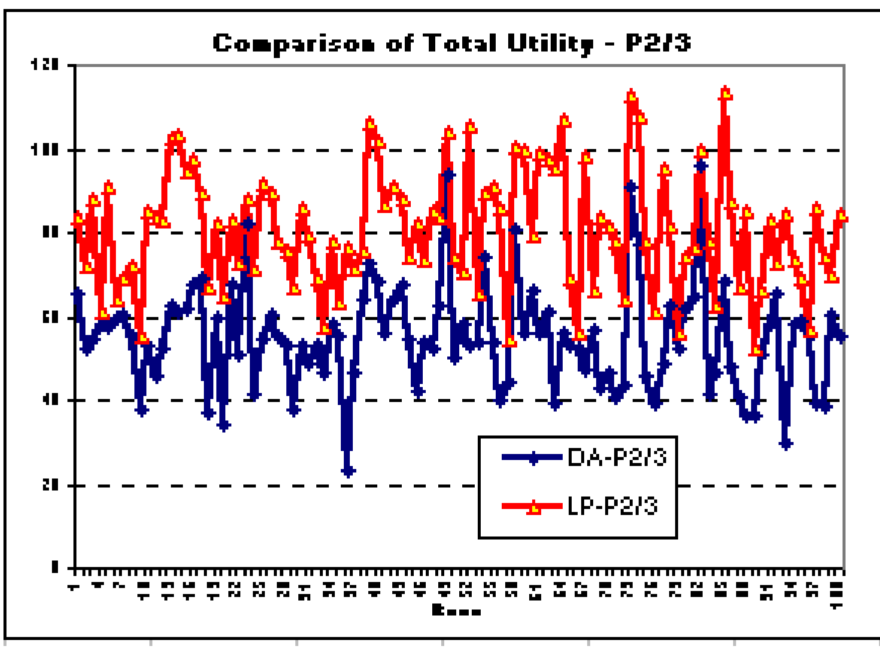

Figure 29. Total Utility of Matches (P2/3 billets)

Table 15. Two-Sample t-test on Means Assuming Unequal Variances (Total Utility)

	P1 Billets		***P2/3 Billets***	
	DA	***LP***	***DA***	***LP***
Mean	59.477	65.274	54.982	80.757
Variance	41.670	21.338	163.518	206.122
Observations	100	100	100	100
Hypothesized Mean Difference	0		0	
df	179		195	
t Stat	-7.304		-13.406	
P(T<=t) one-tail	4.428E-12		8.976E-30	
t Critical one-tail	1.653		1.653	
P(T<=t) two-tail	8.857E-12		1.795E-29	
t Critical two-tail	1.973		1.972	

Because the LP algorithm results in a higher percentage of successful matches, it is not a surprise that the total utility under this method consistently returns a higher score than those from the DA method. An alternative measure of this qualitative factor is to examine the *average utility* instead of *total utility* score. The average utility is computed using the *total utility* divided by the number of successful matches. Instead of comparing the average utility, comparison is done with a measure of proportion called the *percent average total utility* derived by dividing the average utility by the maximum utility of 5. The percent average total utility will also facilitate computing the composite score below.

Figure 30 compares the percent average total utility for sailors between DA and LP for the matching process involving P1 billets. There is no significant difference between the percent average total utility between the DA and LP methods at the 0.05 level of significance (Table 16).

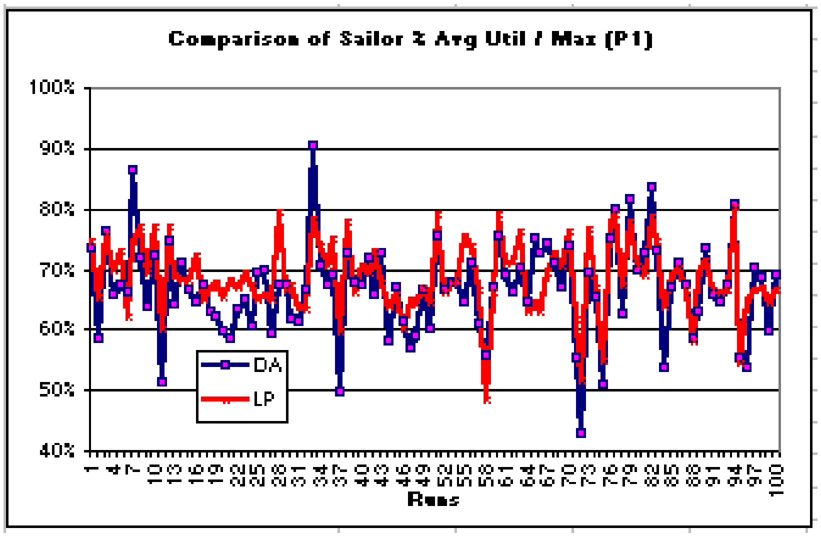

Figure 30. Sailor Percent Average Utility (P1 billets)

For the command's percent average total utility, the LP outcomes are more favorable compared to DA (see Figure 31). The LP method gives a 4.6% higher percent average total utility than the DA method at the 0.05 significance level (see Table 16).

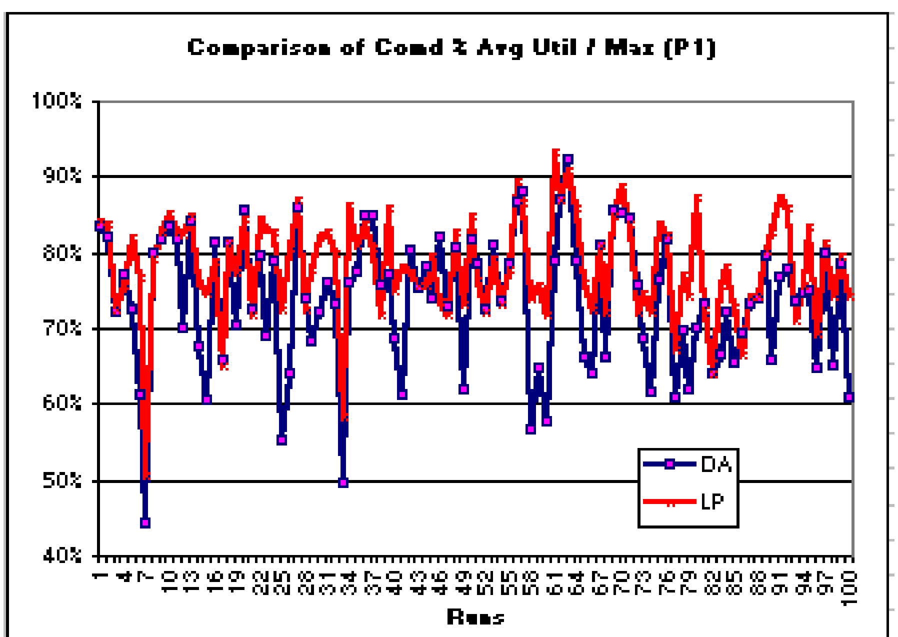

Figure 31. Command Percent Average Utility (P1 billets)

Table 16. Two-Sample t-test on Means for P1 Billets Assuming Unequal Variances (Percent Average Utility)

	Sailor		***Command***	
	DA	***LP***	***DA***	***LP***
Mean	0.668	0.687	0.736	0.782
Variance	0.00594	0.00379	0.00801	0.00470
Observations	100	100	100	100
Hypothesized Mean Difference	0		0	
df	189		185	
t Stat	-1.954		-4.062	
P(T<=t) one-tail	0.0261		3.6E-05	
t Critical one-tail	1.653		1.653	
P(T<=t) two-tail	0.0522		7.19E-05	
t Critical two-tail	1.973		1.973	

For the matching outcomes involving P2/3 billets (see Figures 32 and 33), there is no difference between the DA and LP for percent average total utility for sailors but there is a significant difference at the 0.05 level for percent average total utility for commands. For the commands, the percent average total utility is 28.6% higher for DA than for LP (Table 17).

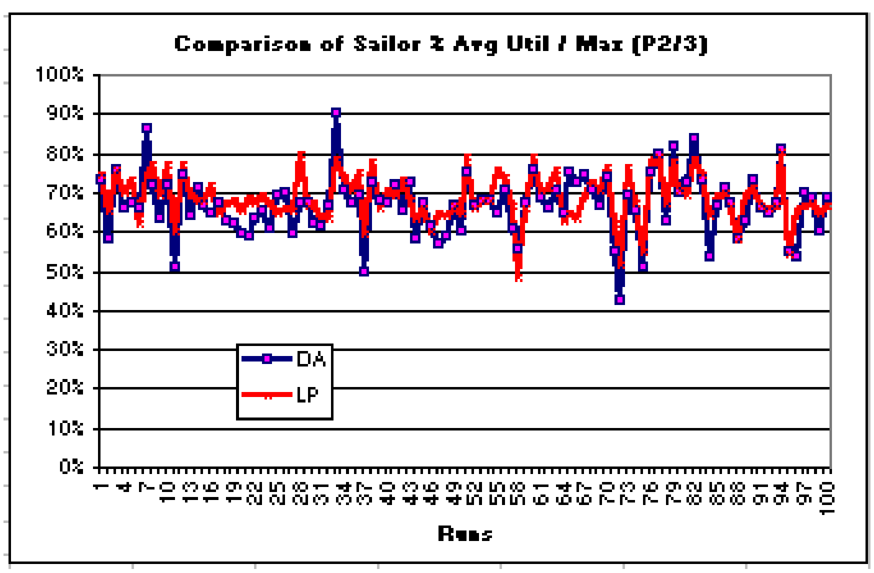

Figure 32. Comparison of Sailor Percent Average Utility (P2/3)

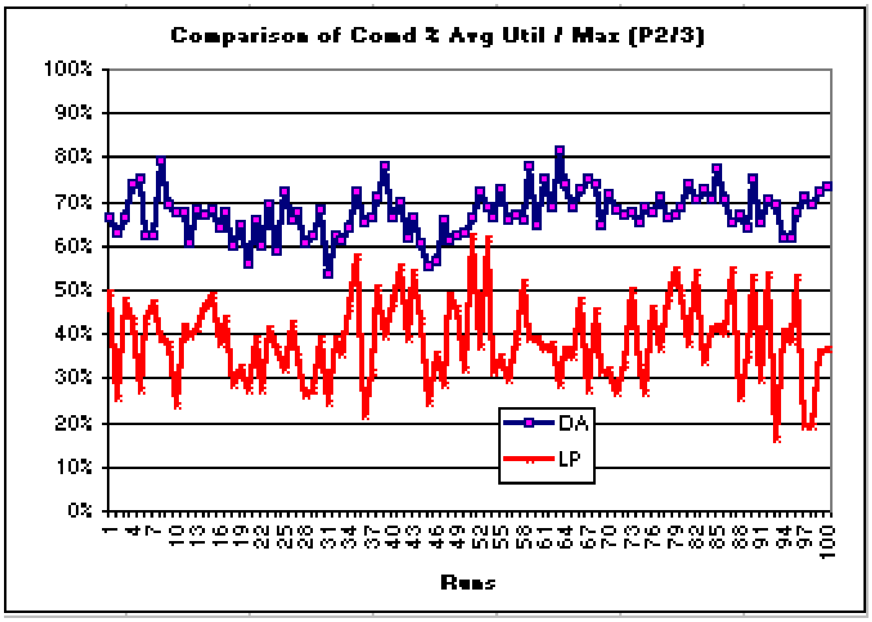

Figure 33. Comparison of Command Percent Average Utility (P2/3)

Table 17. Two-Sample t-test on Means for P2/3 Billets Assuming Unequal Variances (Percent Average Utility)

	Sailor		***Command***	
	DA	***LP***	***DA***	***LP***
Mean	0.851	0.846	0.673	0.387
Variance	0.00221	0.00193	0.00278	0.00971
Observations	100	100	100	100
Hypothesized Mean Difference	0		0	
df	197		151	
t Stat	0.794		25.653	
P(T<=t) one-tail	0.214		3.276E-57	
t Critical one-tail	1.653		1.655	
P(T<=t) two-tail	0.428		6.552E-57	
t Critical two-tail	1.972		1.976	

There is an important caveat: the simulation results may underestimate the total and average utility from the LP results as the matching pairs under the LP algorithm may include some sailors or billets that do not include their matched billets/sailors within their first five choices. In this case, the utility of the matched sailors/billets are assumed to be zero, although they still would have contributed to their partners' utility in reality. Hence there is a possibility that LP has under performed in the simulation because of design factors.

3. Comparison of Unstable Matches

The LP method appears to perform better in terms of percentage matched whilst the DA is more favorable in the measures of percent average utility, as illustrated in the two sub-sections above. However, the second qualitative measure of unstable matches tends to boost the advantages enjoyed by the DA algorithm. This is because the DA method guarantees stable matching, which is not the case under the LP algorithm. Figure 34 shows the proportion of unstable matches for both methods with P1 and P2/3 billets. Runs with P2/3 billets have a much lower tendency for unstable matching due to the larger number of billets available which improves the chances of getting a desired match.

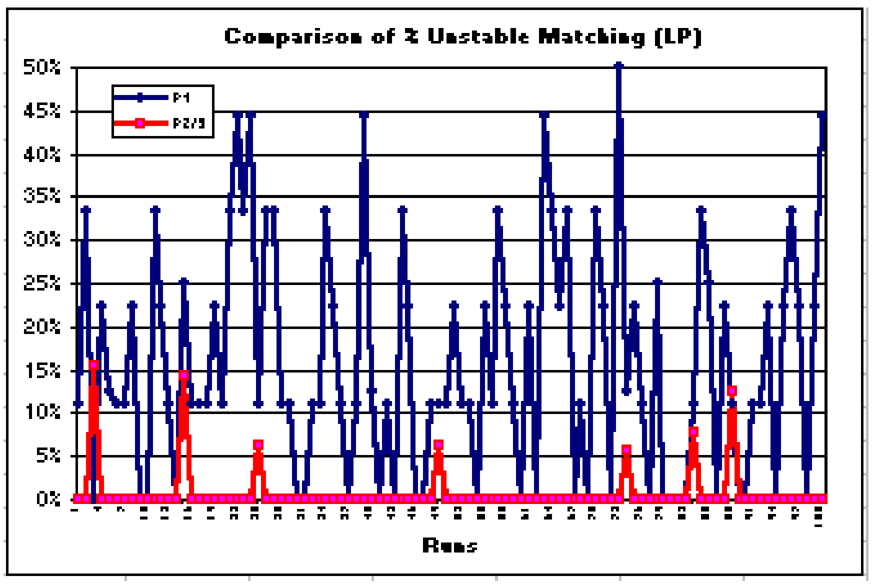

Figure 34. Percent Unstable Matches

C. COMPARISON OF COMPOSITE PERFORMANCE MEASURES

Section B illustrates the trade-offs between the measures of quantity and the measures of quality from the simulation runs. It is not sufficient to evaluate the relative strengths and weaknesses of the DA and LP method on any single measure separately, as DA is better in generating stable matches and generally higher average utility, whilst the LP is better at generating higher percentage of matches. As such, a composite performance

measure that incorporates all the measures is needed for comparison. The following discusses two possible composite performance measures.

1. Composite Performance Measure 1

The first composite score comprises three factors: (a) percent average utility; (b) percent of unstable matches; and (c) percentage of successful matches. The computation is represented by the following equation where equal weights, W1, W2 and W3, are assigned to each of the components:

$$CPM1 = \textit{percent average total utility}^{w1} * \textit{percent stable matches}^{w2} * \textit{percent successful matches}^{w3}$$

As the number of P1 billets is much smaller than the number sailors, the percent of successful matches uses the total number of possible matches as a denominator rather than using the total number of sailors as the denominator (Percent Successful Matches = No. of Sailors Matched / No. of Possible Matches). For P2/3 billets, the percent of successful matches is based on the average of both sailor's and command's percent of successful matches.

Figure 35 shows the difference between the DA and LP algorithms for both the P1 and P2/3 billets. From this chart, there appears to be little difference between the DA and LP methods for matching P1 billets. The LP method, however, scores higher for the matching process involving P2/3 billets. The graphical result is confirmed by the t-tests done at the 0.05 level shown in Table 18. Hence, the LP algorithm is superior when there are a large number of billets available for matching in comparison to the number of sailors. For smaller billet to sailor ratios, the results for DA and LP are similar.

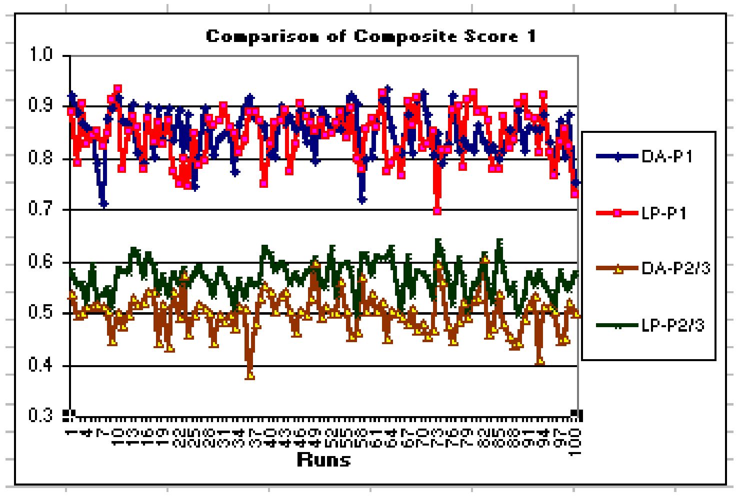

Figure 35. Composite Performance Measure 1 (P1 & P2/3 billets)

Table 18. Two-Sample t-test on Means for Composite Performance Measure 1 Assuming Unequal Variances

	P1 Billets		***P2 Billets***	
	DA	***LP***	***DA***	***LP***
Mean	0.851	0.845	0.501	0.570
Variance	0.00216	0.00249	0.00148	0.00113
Observations	100	100	100	100
Hypothesized Mean Difference	0		0	
df	197		194	
t Stat	0.809		-13.416	
P(T<=t) one-tail	0.210		9.18E-30	
t Critical one-tail	1.653		1.653	
P(T<=t) two-tail	0.420		1.84E-29	
t Critical two-tail	1.972		1.972	

2. Composite Performance Measure 2

The second composite performance measure is developed to cater to the possible overlap between the two measures of percent average total utility and percent successful matches in CPM1, that may have skewed the result in favor of the method that is strong in both these measures. Composite Performance Measure 2 comprises two factors: (a) percent average total utility, (b) percent of stable matches. The computation is represented by the

following equation where equal weights, W1 and W2, are assigned to each of the components:

$$CPM2 = \textit{percent average total utility}^{w1} * \textit{percent stable matches}^{w2}$$

Figure 36 shows the composite score 2 for the DA and LP algorithms in the matching process involving P1 billets. From this chart, the LP method gives a much more favorable outcome than the DA algorithm. The composite score derived for the LP method is about twice that of the DA algorithm and is confirmed by the t-test performed at the 0.05 level (see Table 19).

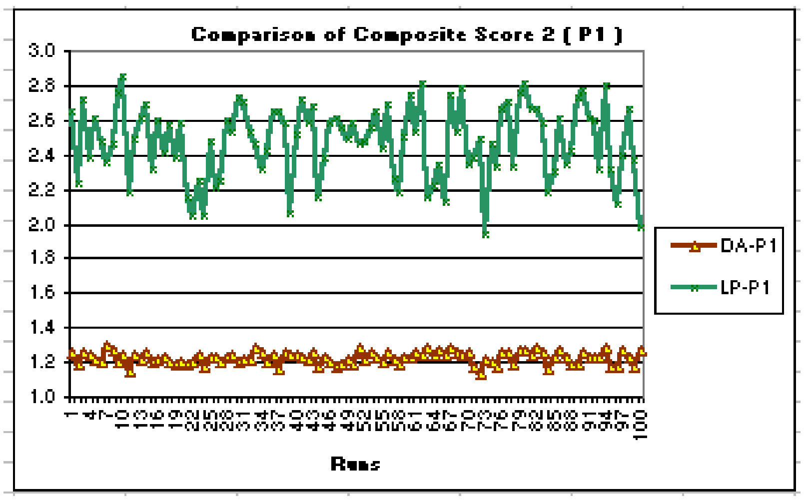

Figure 36. Composite Performance Measure 2 (P1 billets)

The same level of difference in composite score 2 between the DA and LP algorithms is repeated for the process involving P2/3 billets (see Figure 37) and is again confirmed by the t-test performed at the 0.05 level (see Table 19).

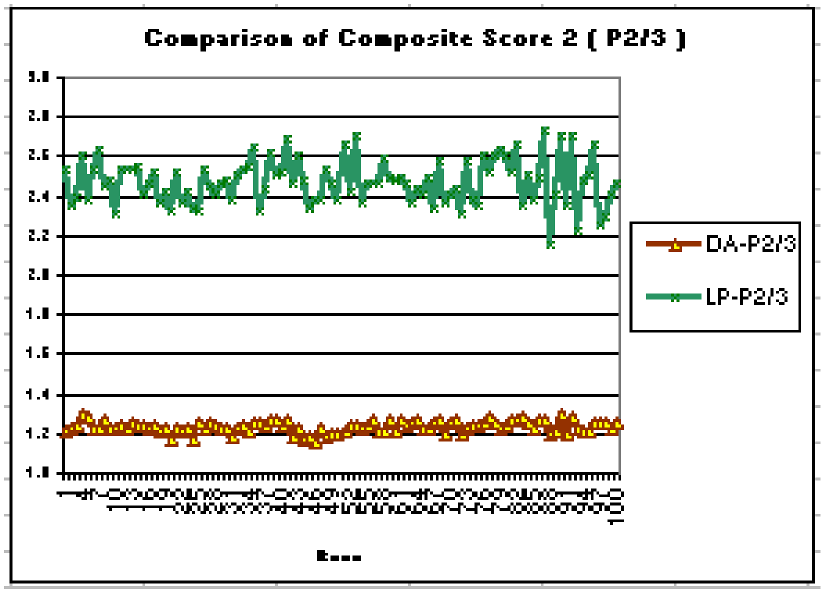

Figure 37. Composite Performance Measure 2 (P2/3 billets)

Table 19. Two-Sample t-test on Means for Composite Performance Measure 2 Assuming Unequal Variances

	P1 Billets		***P2 Billets***	
	DA	***LP***	***DA***	***LP***
Mean	1.225	2.477	1.234	2.471
Variance	0.00119	0.0455	0.0008	0.01303
Observations	100	100	100	100
Hypothesized Mean Difference	0		0	
df	104		111	
t Stat	-57.929		-105.157	
P(T<=t) one-tail	2.84E-81		2.7E-113	
t Critical one-tail	1.660		1.659	
P(T<=t) two-tail	5.68E-81		5.4E-113	
t Critical two-tail	1.983		1.983	

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VI. CONCLUSION AND RECOMMENDATIONS

A. CONCLUSION

The thesis sets out to examine the relative performance of the Deferred Acceptance (DA) and the Linear Programming (LP) algorithm when applied to two-sided matching. The two methods are the principal methods used in the market for two-sided matching processes. The comparison was done through an adapted simulation engine in four parts: (1) the generation of sailor and billet profiles using actual sailor and billet data for a particular community in the US Navy, the AS community, (2) the generation of preference lists for both the sailor and the billet for each other based on preference factors determined previously via survey, (3) the simulation of matches for sailors and billets via both the DA and LP methods, (4) the generation of performance measures for comparison between the DA and LP methods.

One hundred simulation runs, equivalent to one hundred requisition cycles, or about four years' worth of requisitions, were conducted on NEDSim (Navy Enlisted Distribution Simulator). The outcomes from the DA and the LP algorithms were then compared using both quantitative measures (percent matches), qualitative measures (average utility score and stable matches) and composite measures. Two alternative composite scores were developed to incorporate the trade-offs between the individual measures. All the measures were weighted equally.

The results of the comparison were as follows:

- The LP method outperforms the DA algorithm in the area of percent matches for both P1 and P2/3 billets as well as for both the command and the sailor.

- There was no difference between the LP and the DA methods in terms of average utility for sailors in P1 and P2/3 billets but LP performed marginally

better for commands (4.6%) in P1 billets and DA performed significantly better for commands in P2/3 billets (28.6%). However, the performance for LP could be underestimated due to limitations in the simulation model.

- DA was significantly better in stable matches for P1 billets and slightly better for P2/3 billets.

- LP was superior to DA when the measures were combined into equally weighted composite scores.

B. RECOMMENDATIONS

Based on the results of the simulation comparison, the US Navy may want to seriously consider adopting the LP method as the two-sided matching mechanism for its sailors. The decision will still largely depend on the relative importance the organization places on each component of the composite score currently equally weighted, and on the priority of assigning sailor only to billets in their preference list (as in the DA method but not LP). Even though it is ideal to eliminate unstable matching, this factor may not be so desirable that it overwrites the alternative objective of achieving more matches. The results indicate that the proportion of unstable matching under the LP method may be manageable. Furthermore, private arrangements between sailors and commands out of the system may not be as prevalent as in other private organizations in view of the military culture and the size of the organization which makes it difficult for many people to be identified individually.

C. AREAS FOR FURTHER RESEARCH

There are a few possible areas for further research. The possible areas are in applying the model to more communities, improving the simulation model and verification of the results on actual sailor and command input. The specific areas that could be studied include:

- Examining the applicability of the simulation to another community or to a more general population to see if the results can be generalized.

- Including more sailor and command preference factors in the simulation as currently only those factors favored by at least half of the survey respondents are included.

- Including other preference factors that may be important but were not reflected in the previous survey that was done.

- Refining the simulation program to factor actual billet and sailor profiles from different communities to generate the utility scores and preferences.

- Conducting an actual evaluation experiment of DA versus the LP method of matching by using actual sailor and billet choices and selection scores.

- Conduct more rigorous analysis of the impact of varying the sailor:billet ratio as well as the P1:P2/3 ratio on unstable matching.

- Restricting the LP mechanism to conduct matches only on those billets and commands that have expressed a choice for each other.

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